ISS Comparison of Schemes

R. B. Palmer (BNL)
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Subjects I will discuss

1. RF Systems
2. Longitudinal Capture
3. Transverse Capture and Cooling
4. Performance by muons/initial pions
5. Performance by muon decays per year

Reservations

- Using mixture of published and estimated numbers
- Aim for qualitative understanding
- Not exact performances

Studies Used

1. At 5 MHz by Japanese Group (Nufact J Report)
2. At 44 and 88 MHz by CERN Group (CERN reports nf20, nf34, nf87)
3. Another study at CERN using only 88 MHz was not included due to lack of sufficient available information. It had similar performance to the 44 and 88 MHz case.
4. At 201 MHz by the US Collaboration using Induction phase rotation (Feasibility Study 2)
5. At 201 MHz by the US Collaboration using bunched beam phase rotation (Study 2a)

Method to Compare performances

- Study Muons out per Captured Pion (at 1m in capture channel)
- avoids uncertainties in production
- all systems have similar capture efficiency to this point
- Assume orthogonality between transverse and longitudinal phase spaces

$$\eta_{\mathrm{front-end}} = \eta_{||} \quad \eta_{\perp}$$

$$\frac{\mathrm{Final\ Muons}}{\mathrm{Captured\ Pions}} = \eta_{\mathrm{all}} = \eta_{||} \quad \eta_{\perp} \quad \eta_{\mathrm{accel}} \quad n_{\mathrm{signs}}$$

- Include decay losses in phase rotation in $\eta_{||}$
- Include decay losses in cooling in $\eta_{\perp}$
- Estimate $\eta_{||}$ from published information
- Estimate $\eta_{\perp}$ without cooling from our simulations
- Estimate $\eta_{\perp}$ with cooling from published $\eta_{\mathrm{front-end}}$ and $\eta_{||}$
- n_{signs} is one for all except US Neuffer Phase rotation

- I will refer to the three projects by the frequencies: 5, 88, and 201 MHz

Longitudinal Capture Phase Space

Problem is to match initial muon longitudinal phase space into RF bucket

- Initial Longitudinal Acceptance $A_{\|}$ of all muons: $$A_{\|} \;=\; \beta\gamma\,\frac{\Delta_E}{E}\,c\Delta_t$$

  $c\,\sigma_t$ from decay $\approx$ 1 m, $\Delta ct = 2\,\sigma_t$ ΔE/E=100%, and $\beta\gamma = 2$:

  $$\boxed{A_{\|} \;=\; 2\times 100\% \times 2m \;=\; 4(\mathrm{m}) \;=\; 1.3\ (\mathrm{eV\ sec})}$$

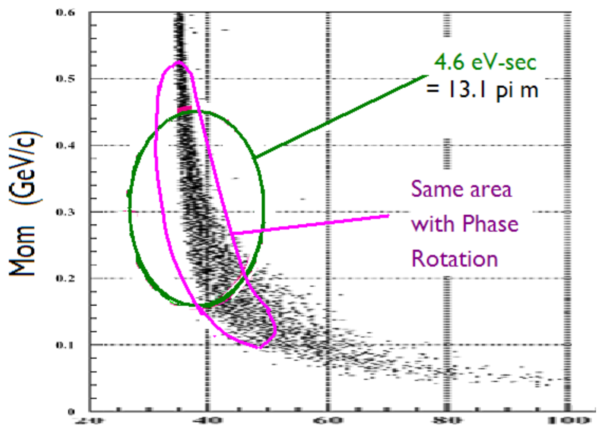
- Bucket areas A_{bucket}:

  If limited by $\Delta p/p$, then: $$A_{\mathrm{bucket}} \;\propto\; \left(\frac{\Delta}{p}\right)^2 \sqrt{\frac{1}{f\ \mathcal{E}\ \cos\phi}}$$

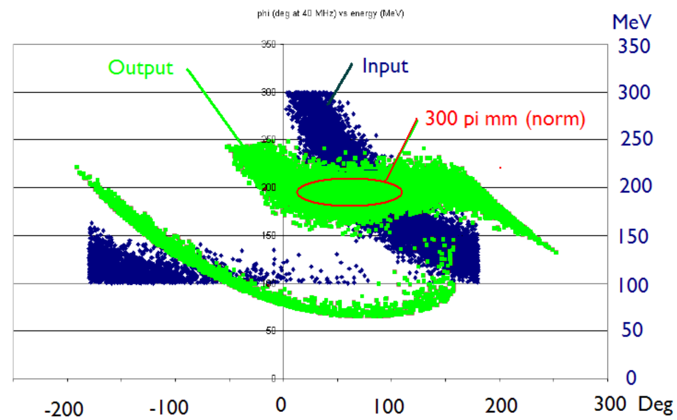
f (MHz)	n bunches	$\mathcal{E}$ (MV/m)	$\Delta p/p$	$A_{\|}$ (pi m)	$A_{bucket}/A_{\|}$
5	1	1	50%	13	3.2
88	1	4	12%	0.3	0.08
201	50	11	22%	0.15×50=7.5	1.8

- 5 MHz and 201 MHz have enough acceptance to capture entire production
- 44/88 MHz lacks longitudinal acceptance
- To best match into bucket requires "Phase Rotation"

Phase Rotation



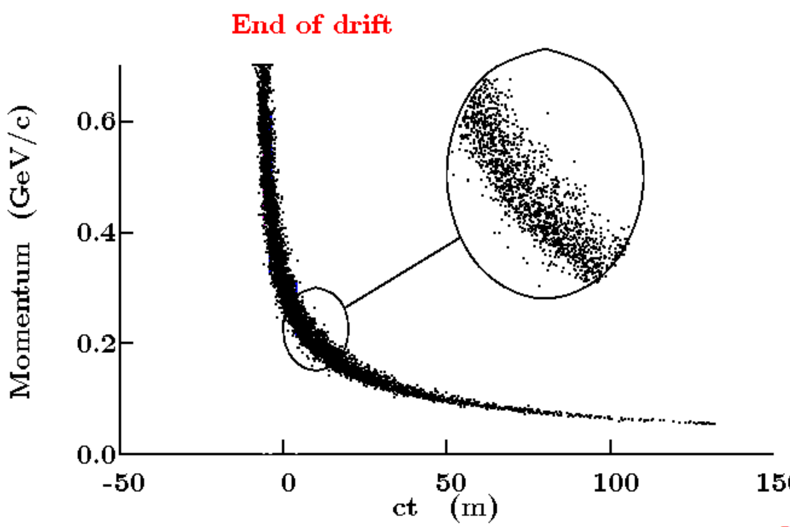
5 MHz Direct Capture

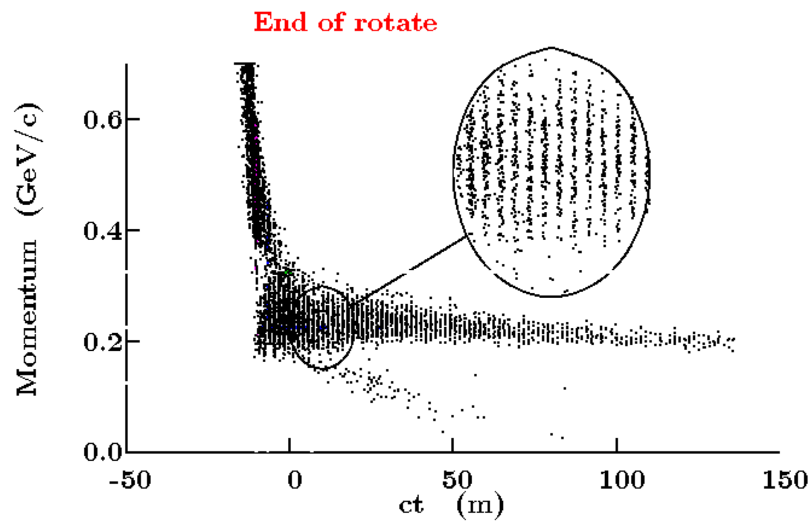


44/88 MHz

Direct into FFAG Lattice +/- 50% $\Delta p/p$

30 m drift, 30 m, 44 MHz Rotation

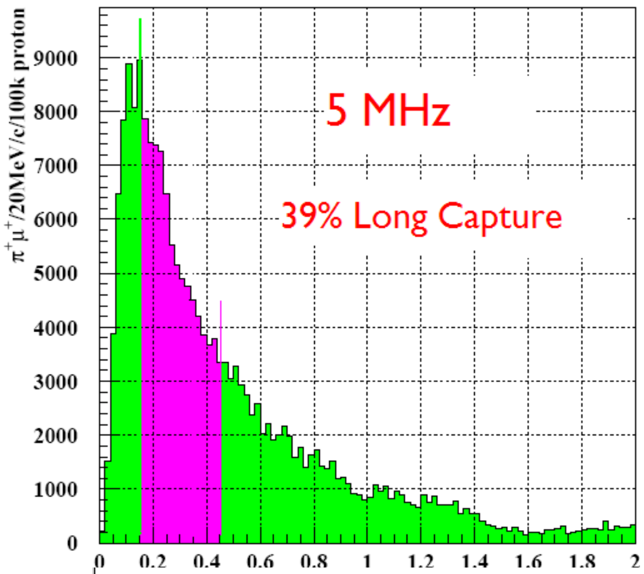


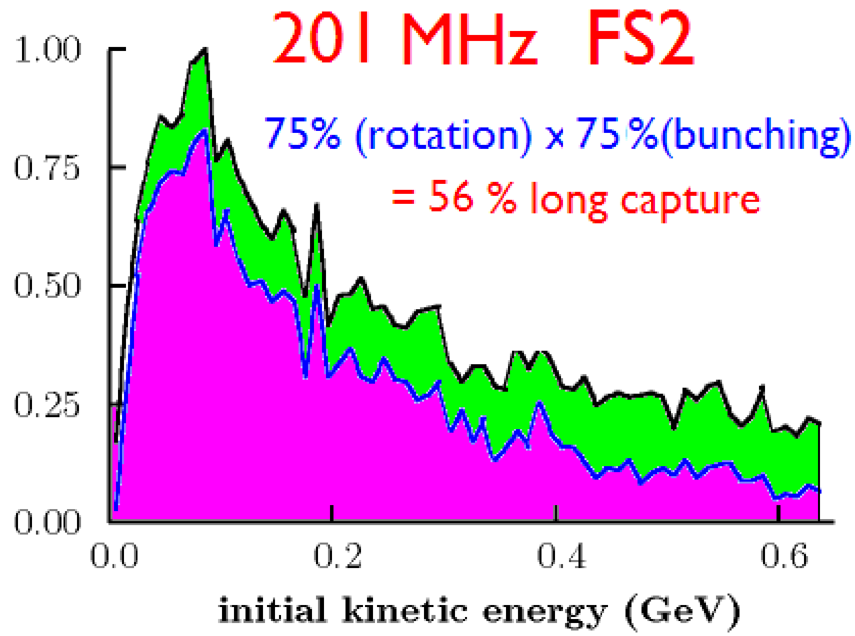


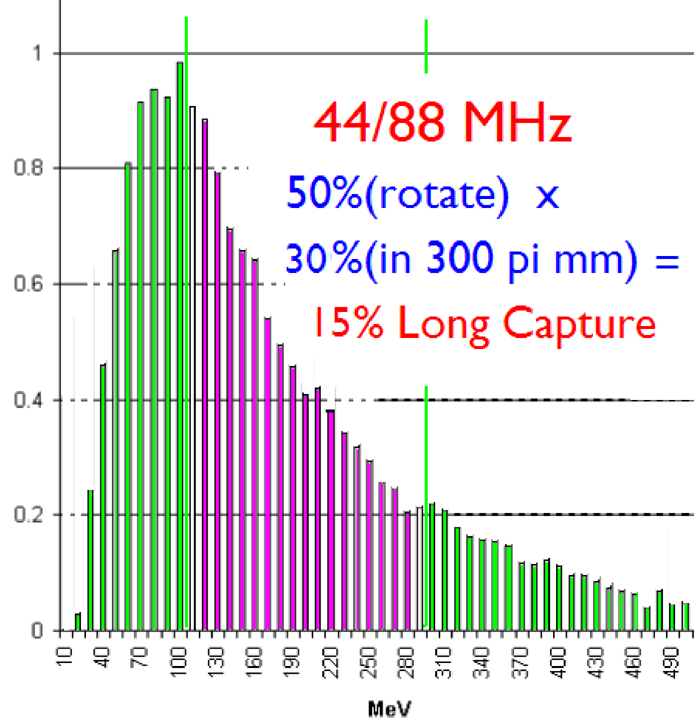
200 MHz Study 2a

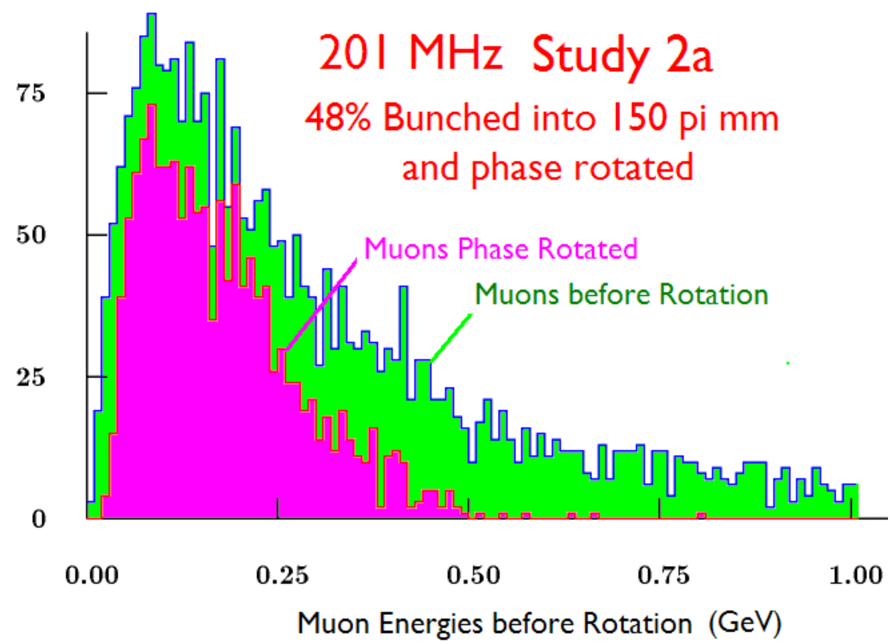
201 MHz S2a: 100 m drift, 40 m 300-230 MHz buncher , 54 m 230-201 MHz Rotator

Longitudinal Capture Efficiency including decays in rotation









Longitudinal Capture Efficiencies $\eta_{\parallel}$

Case	Rotated	% in $A_{\parallel}$	$\eta_{\parallel}$	cf $A_{\parallel}/A_{prod}$
5 MHz		39%	39%	3.2
(5 MHz + Phase Rotation)	(≈60%)		(≈60%)	3.2
44/88 MHz	50%	(30%)	(15%)	8%
201 MHz FS2	75%	75%	56%	1.8
201 MHz Study 2a	48%		48%	1.8

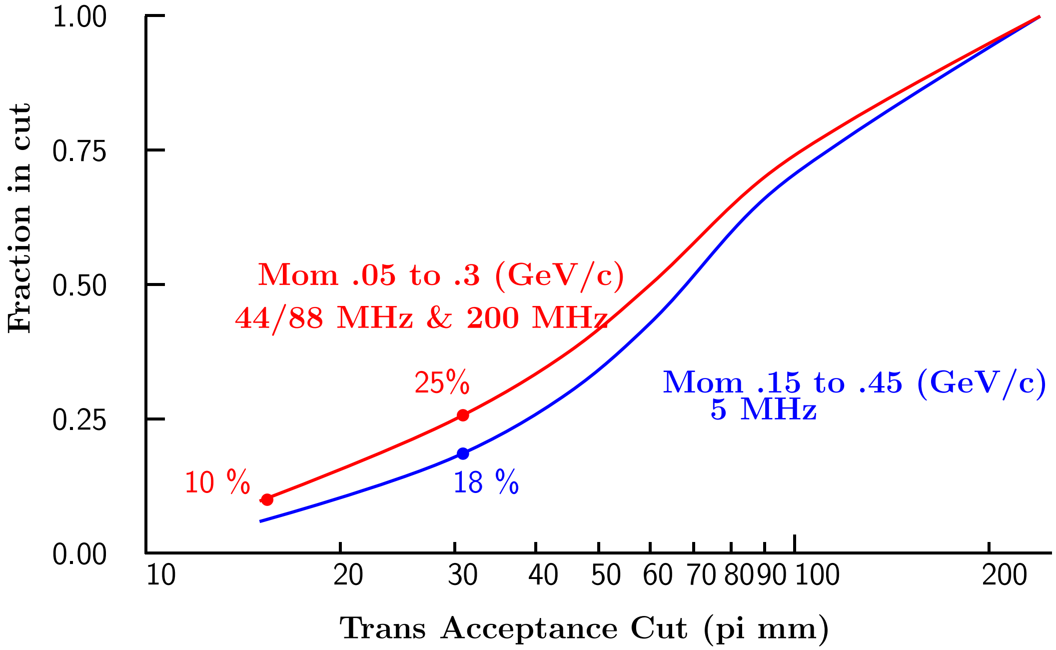
- Rotation would help 5 MHz scheme, but long and expensive
  efficiency is only an estimate: rotation not designed
- Low 44/88 MHz efficiency due to small $A_{\rm bucket}/A_{\rm Production}$
- 44 or 88 MHz could probably rotate to multiple bunches, gaining ≈ 5 and both signs but proton bunches must then be well apart

Transverse Acceptance ($\eta_\perp$) if no cooling

Assume trans momentum distributions independent of p energies (true at high E)

But they are certainly dependent on pion total momenta

Use 24 GeV MARS with mercury



If no cooling

- $\eta_\perp$=18 % 5 MHz (30 pi mm)
- $\eta_\perp$=10 % 44/88 MHz (15 pi mm)
- $\eta_\perp$=25 % 201 MHz (30 pi mm)

- Less accepted at higher total momenta

Transverse Acceptance ($\eta_\perp$) with Cooling

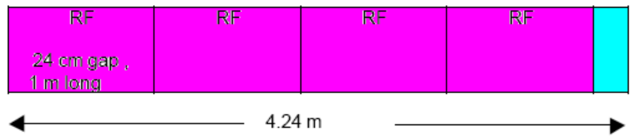
- Use published $\eta_{\text{front-end}}$ and above $\eta_{||}$

Cooling

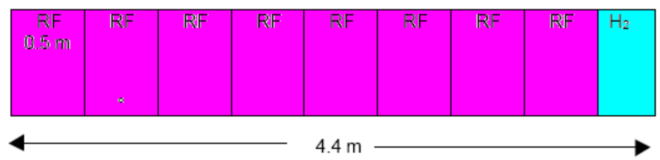
5 MHz   Possible cooling with hydrogen gas in first FFAG

- If acceptance of this ring not greater than later rings then there is no gain
- lowest mom ring is hardest to get large transverse acceptance
- so no expected performance gain, but a cost reduction if later rings need less acceptance

44/88 MHz



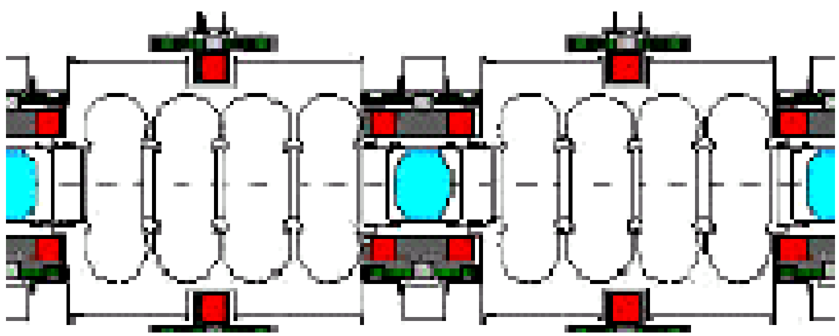
44 MHz



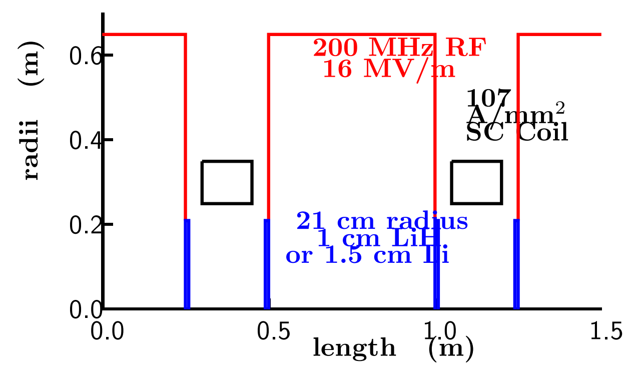
88 MHz

1) 46 m 44 MHz Cooling   2) 32 m Acceleration   3) 112 m 88 MHz Cooling

201 MHz



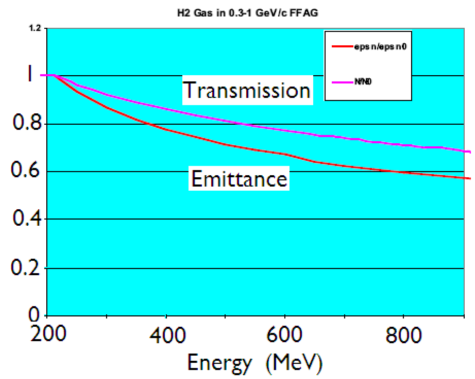
FS2 Cooling



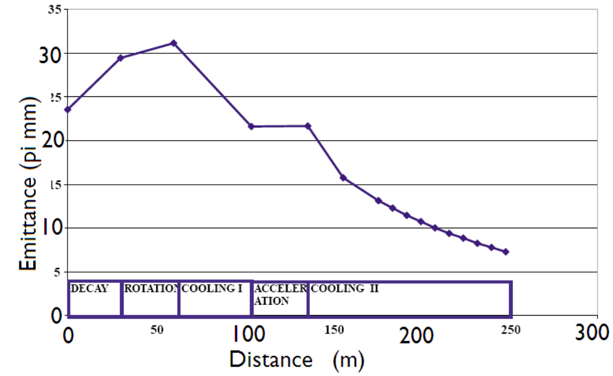
S2a Cooling - same RF

Cooling Performances

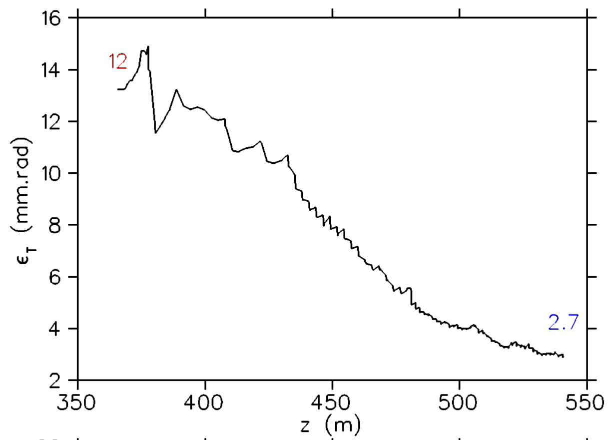
5 MHz

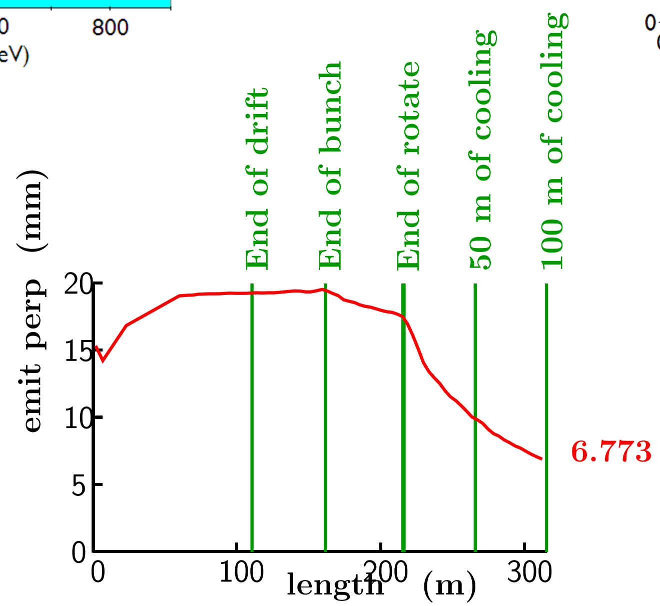


44/88 MHz



201 MHz





- Cooling not part of 5 MHz baseline, I expect no intensity gain
- Larger gain if $A_\perp$ small
- 44/88 MHz has most cooling

	ϵ_1 pi mm	ϵ_2 pi mm	ϵ_2/ϵ_1	$A_\perp$ pi mm	$\eta_\perp$ cool	$\eta_\perp$ no cool	factor
5 MHz	?	?	0.55	30	.18	$\approx$ .18	$\approx$ 1
44/88 MHz	30	6	0.2	15	.67	.10	6.7
201 MHz FS2	12	2.7	.22	15	.38	.10	3.8
201 MHz S2a	20	6.8	0.34	30	.42	.24	1.7

Acceleration

For constant accelerating gradient $\mathcal{E}$:

$$\eta_{\mathrm{accel}} = \left(\frac{n_2}{n_1}\right) = \left(\frac{E_1}{E_2}\right)^{\frac{m_\mu}{c\tau_\mu}\mathcal{E}}$$

	E_1 GeV	E_2 GeV	f MHz	$\mathcal{E}$ MeV/m	η_{accel}
5 MHz	0.21	20	5	1/.75	0.5/ 0.36
44/88 MHz	0.20	20	88 & 176	1.8	0.65
201 MHz	0.13	20	201	4.0	0.81

- 5 MHz had earlier give $\mathcal{E} = 1$ (MeV/c), but have recently lowered it to 0.75 MV/m
- The figures for 44/88 and 201 MHz are "effective", the real values are not constant
- Actual gradients in 44/88 MHz Scheme are 4 and 10 MeV/m at 88 and 176 MHz
- Actual gradient in the 201 MHz schemes is 17 MV/m superconducting
- The advantage of the higher gradient at the higher frequencies is apparent

Overall Performance Parentheses on estimated values

case	Cool?	$A_\perp$ pi mm	$\eta_{\parallel}$	$\eta_\perp$	η_{front}	η_{accel} %	n_{signs}	η_{all} %
5 MHz	no	30	.39	(0.18)	16 (7)	0.36	1	6[1] (2.5)
44/88 MHz	yes	15	(0.15)	[0.67][2]	10	0.66	1	6.6[3]
44/88 MHz	no	30	(0.15)	(0.24)	(3.6)	0.66	1	(2.4)
201 MHz FS2	yes	15	0.56	0.38	21	0.81	1	17
201 MHz FS2	no	30	0.56	0.24	13	0.81	1	11
201 MHz S2a	yes	30	0.48	0.42	20	0.81[4]	2	33
201 MHz S2a	no	30	0.48	0.24	12	0.81[4]	2	19

- 5 MHz's efficiency without cooling equals 44/88's with cooling, showing the advantage of 5 MHz larger acceptance
- We believe 5 MHz's efficiency could be raised by adding phase rotation and cooling, but is unlikely to match 201 MHz S2a because of greater decay losses in acceleration, and apparent inability to capture both signs
- 44/88 MHz's performance has the best cooling performance ($\eta_\perp$), but this is offset by its poor longitudinal acceptance ($\eta_{\parallel}$)
- 201 MHz S2a's strength comes from the combination of good longitudinal acceptance ($\eta_{\parallel}$) and the capture of both signs

Notes for previous table

1. This value is obtained from quoted 0.3 captured muons per proton, 3.5% MARS captured pions per proton GeV, and 0.5 acceleration decay loss. The discrepancy with my estimate is not understood.
2. This is derived from our estimate of $\eta_{\|}$ and the CERN given η_{front} and seems unrealistic
3. From table in CERN note #20
4. Matching loss not included since no such loss in other examples included

The best features

- In 5 MHz Scheme
  - The use of very large accelerator/storage ring acceptance
    Allows reasonable performance without cooling
    Improves performance with cooling
    Adopted by US Study 2a
- In 44/88 MHz Scheme
  - Using many RF cavities before hydrogen absorbers
    Allows use of fewer, but longer absorbers
    Reduces cost
    Reduces effect of windows
  - Most effective cooling scheme
- In 201 MHz S2a Scheme
  - Bunched Beam Phase Rotation
    Allows large initial longitudinal acceptance without low frequency
    Captures both signs

Conclusions

- All designs have particular good ideas
- As presented, only US FS2 achieves the 10^{21} useful decays per year goal
- The use of Neuffer phase rotation is its critical advantage
  Large longitudinal acceptance and both signs
  - This could probably be used at 88 MHz in a CERN like scheme
  - The very low frequency of the Japan scheme does not seem to allow this option
- There is a clear advantage in the higher gradients with the higher frequency
  - The limit to this frequncy appears set by the required iris to transmit the initial transverse acceptance

Possible improvements

- In 5 MHz Scheme
  - Add linear Phase rotation before acceleration
    gain of up to a factor of 2
  - Add linear Cooling (linear because it requires acceptance >> 30 pi mm)
    gain up to about 50%
  - But without both signs it is unlikely to match Study 2a
- In 44/88 MHz Scheme
  - Use Bunched Beam Phase Rotation
    Gain up to about 3 in longitudinal capture and get both signs
  - Increase acceptance of acceleration and use less cooling
    Performance should then approach US Study 2a
  - But the lower accelerating gradients at 88 MHz will remain a disadvantage
- In 201 MHz FS2a
  - More cooling in a lower beta channel would give some improvement,
    but at considerable cost
  - Larger accelerator and storage ring acceptance could give significant improvement but appears impractical

Useful Muon decays per year

- With 4 MW proton power
- Assume captured pions per proton GeV $= .039$ (S2a value)
- 60% straight over circumference for two detectors
- 10^7 seconds per year
- Include 20% loss for matching, injection, etc
- Compare with Lyon NuFact goal of 10^{21} Useful decays per year
- Note this is approximately equiv to Study goal of 2 10^{20} per year with 1 MW p power

case	cooling	trans acc pi mm	signs	mu/pi	mu/year $\times 10^{21}$
5 MHz	no	30	1	0.08	.22
44/88 MHz	yes	15	1	0.066	.24
201 MHz FS2	yes	15	1	0.17	.62
201 MHz S2a	yes	30	2	0.17	1.22
201 MHz S2a	no	30	2	0.09	.72

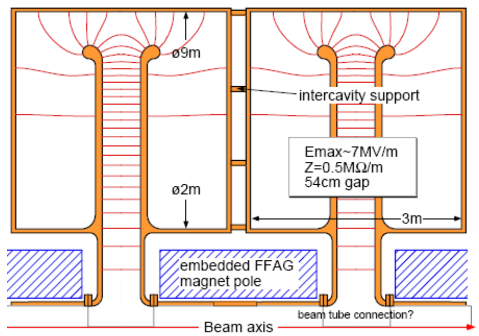
- Only 201 MHz S2a with 2 detectors reaches the 10^{21} goal

RF Frequencies and Systems

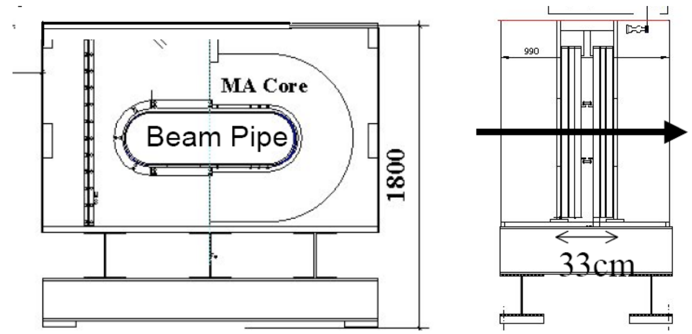
The most significant difference between systems

5 MHz

Studied in Japan



Japan 5 MHz Vacuum RF



Japan Ferrite Loaded RF (cf PRISM)

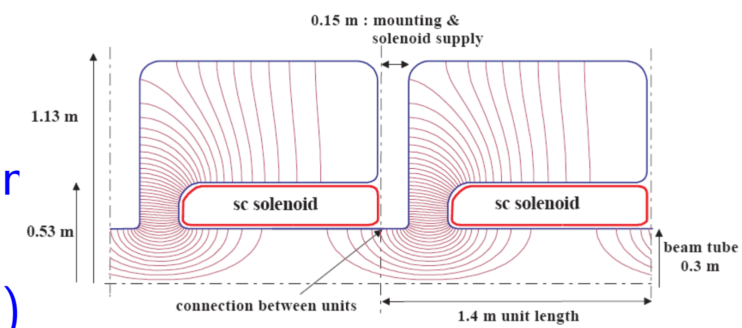
44 and 88 MHz

Studied in CERN

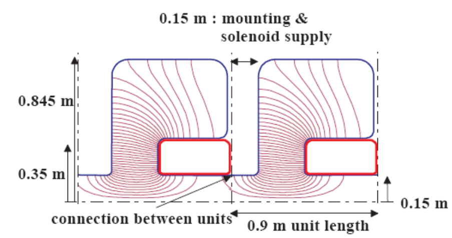
(Preferred all 88 MHz syster

with similar performance,

not sufficiently documented)



44 MHz



88 MHz

201 MHz

Studied in US