Fermi National Accelerator Laboratory

On orientation of accelerating cavities in HCC

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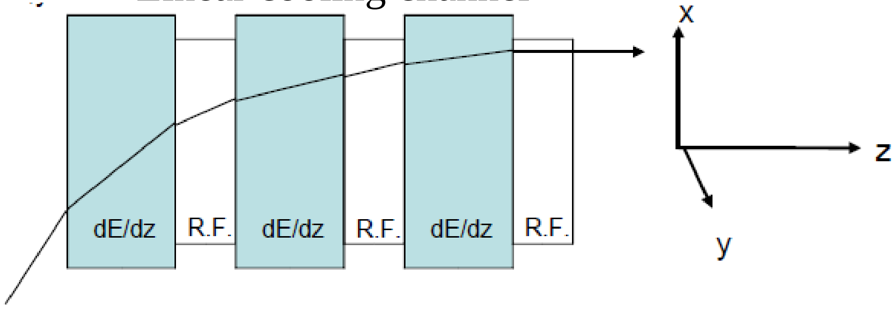
Talk Outline

- **Motivation**
- **CST model of helical dipole.**
- **Analytical model - simplifications. Equations of particle motion**
- **Solving the equations in Mathematica**
- **Ionization losses and energy gain. Equilibrium orbits. RF field amplitude**
- **Impact of RF cavity orientation on cooling.**

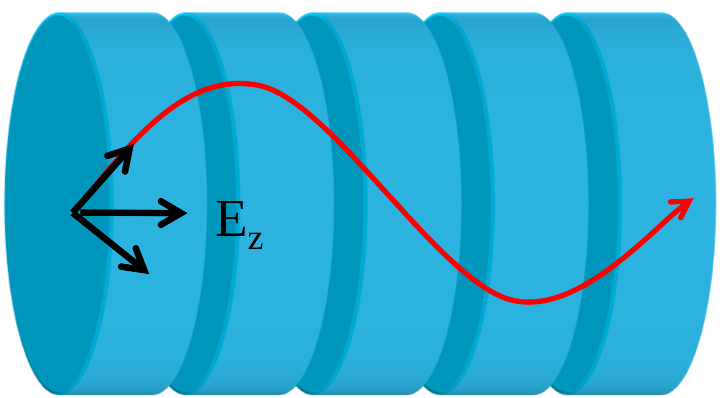
RF cavity orientation

Linear cooling channel



Ionization reduces p_x, p_y and p_z , RF cavities restore p_z , reducing $p_{x,y}$ / p_z and thus transverse emittance.

In HCC equilibrium orbit is a helix, and accelerating field is not collinear with it. Does the transverse kick matter? Should we try to reduce it?



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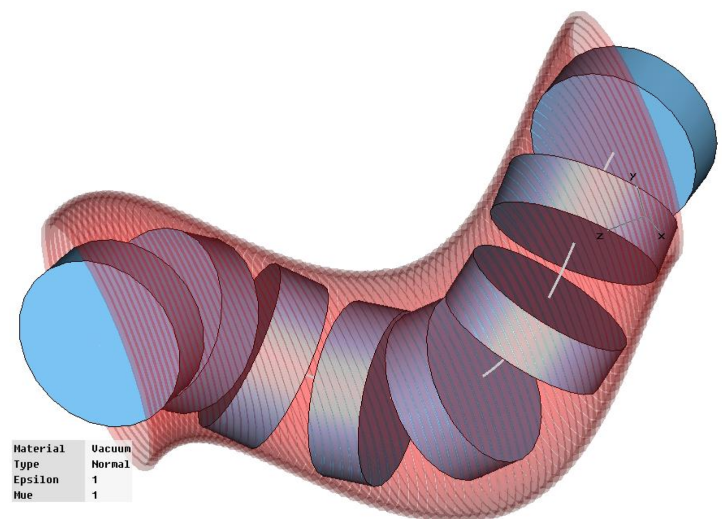
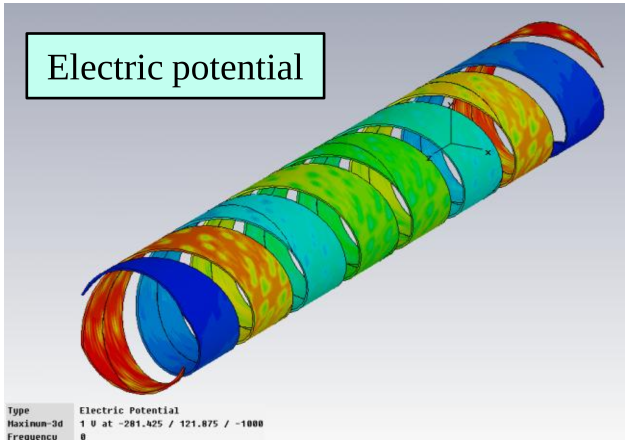
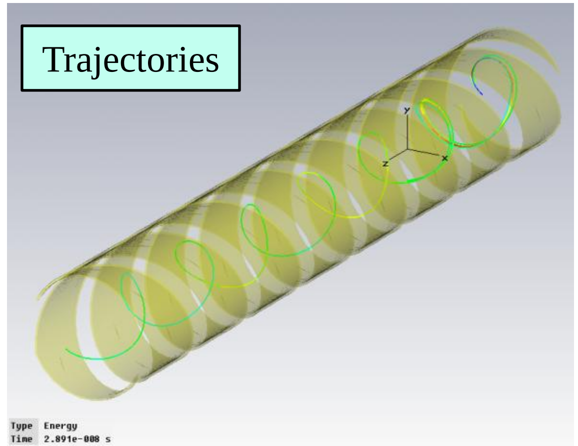


CST helical dipole model





V.Kashikhin et al, EPAC08, WEPD015

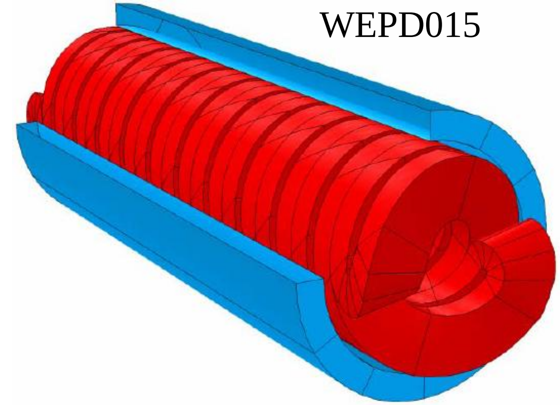
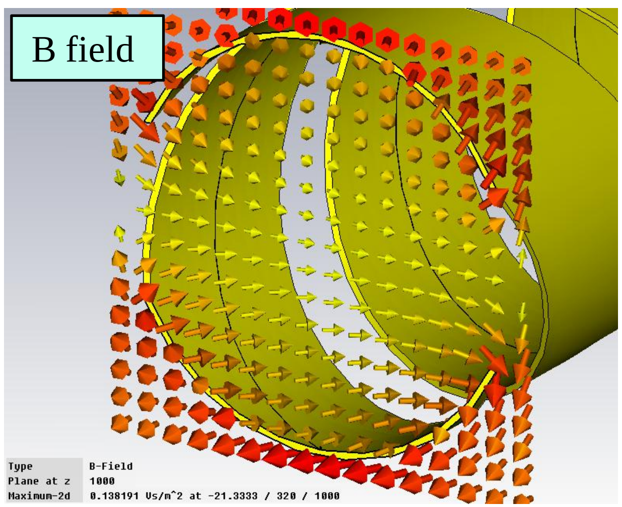
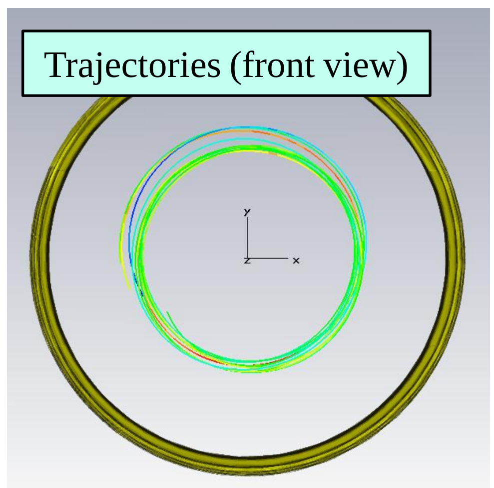


Figure 1: High-field helical dipole with straight solenoid (a quarter removed for clarity).





Period L= 1 m

Bore R = 0.3 m

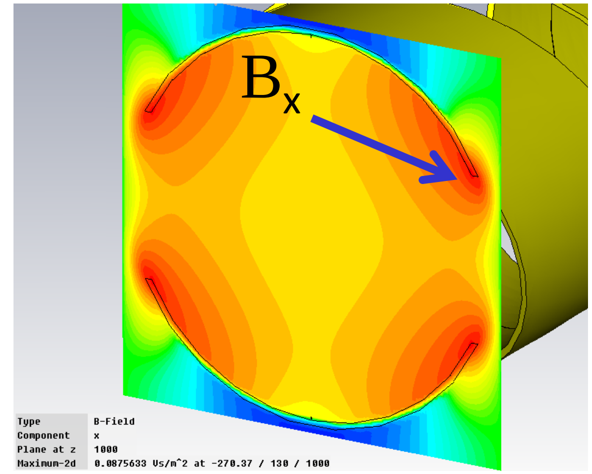
B_s = -6.95 T

B_τ = 1.62 T

Orbit radius = 0.16 m

Analytical description of field

Unfortunately it’s not practical to use CST Particle Studio directly because of number of reasons. Two of them are distorted field at the ends of the magnet and B_z component in the vicinity of equilibrium orbit. The CST model was used to verify the following approximation of helical dipole fields (J. P. Blewett and R. Chasman, J. App. Phys. **48**, 2692(1977)):

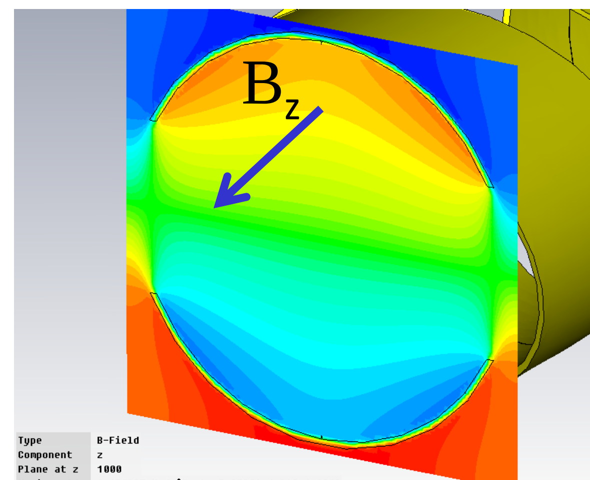


$$B_x \approx -B_0\left\{\left[1+\frac{k^2}{8}\left(3x^2+y^2\right)\right]\sin kz - \frac{k^2}{4}xy\cos kz\right\},$$

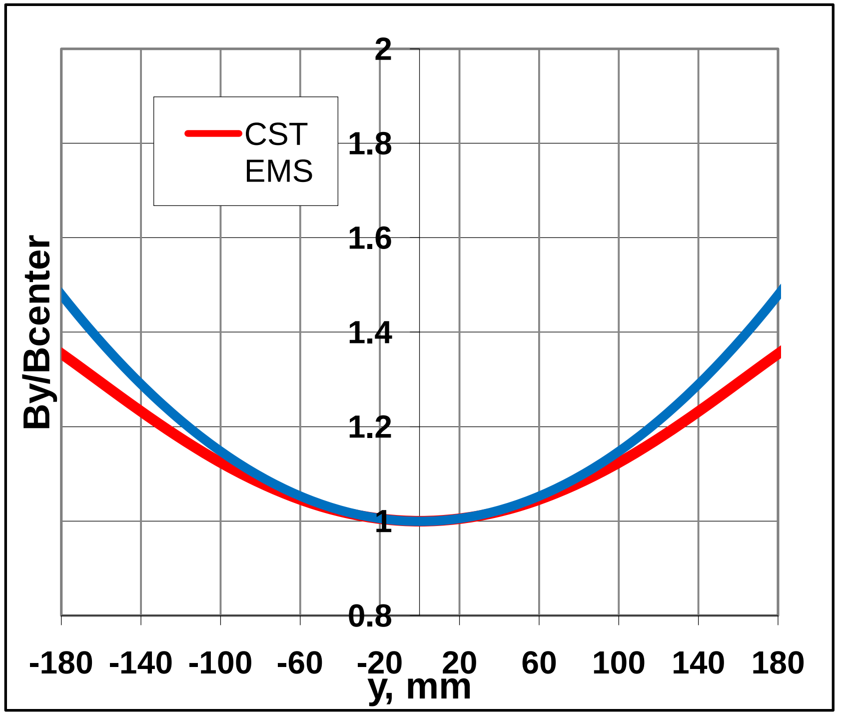
$$B_y \approx B_0\left\{\left[1+\frac{k^2}{8}\left(x^2+3y^2\right)\right]\cos kz - \frac{k^2}{4}xy\sin kz\right\},$$

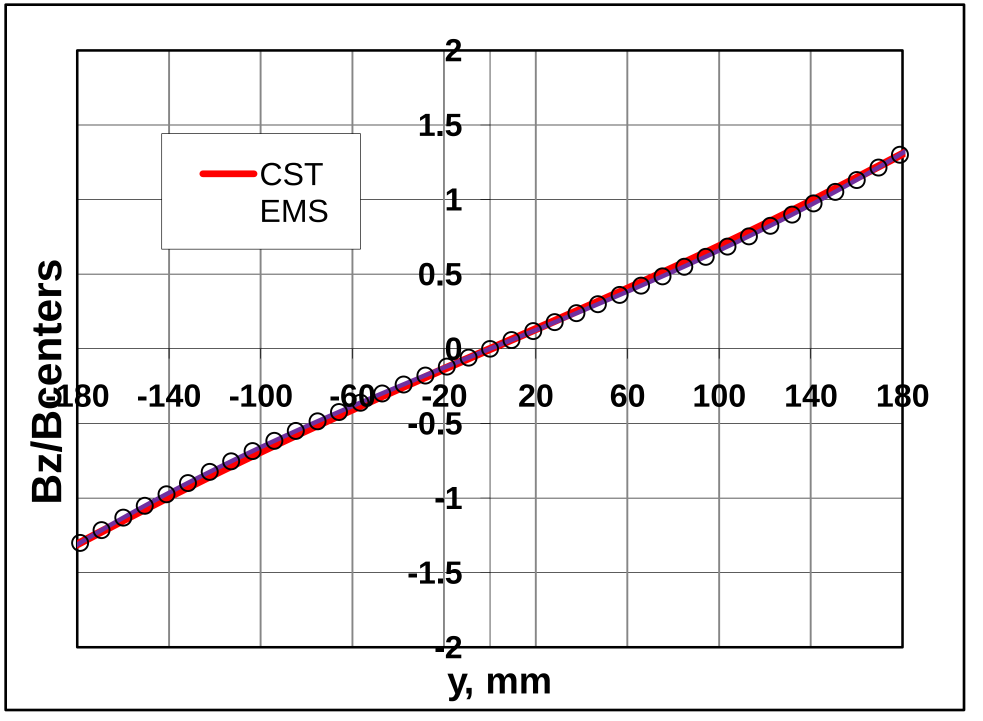
$$B_z \approx -B_0 k\left\{1+\frac{k^2}{8}\left(x^2+y^2\right)\right\}\left[x\cos kz + y\sin kz\right],$$

where $k = 2\pi/L$



Comparison of fields





Equations of motion

In Cartesian coordinates we have for vector components:

$$\frac{d}{dt}(\gamma m\dot{x}) = q\left(E_x + \dot{y}B_z - \dot{z}B_y\right) + \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}}\left(\frac{dW}{ds}\right)$$

$$\frac{d}{dt}(\gamma m\dot{y}) = q\left(E_y + \dot{z}B_x - \dot{x}B_z\right) + \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}}\left(\frac{dW}{ds}\right)$$

$$\frac{d}{dt}(\gamma m\dot{z}) = q\left(E_z + \dot{x}B_y - \dot{y}B_x\right) + \frac{\dot{z}}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}}\left(\frac{dW}{ds}\right)$$

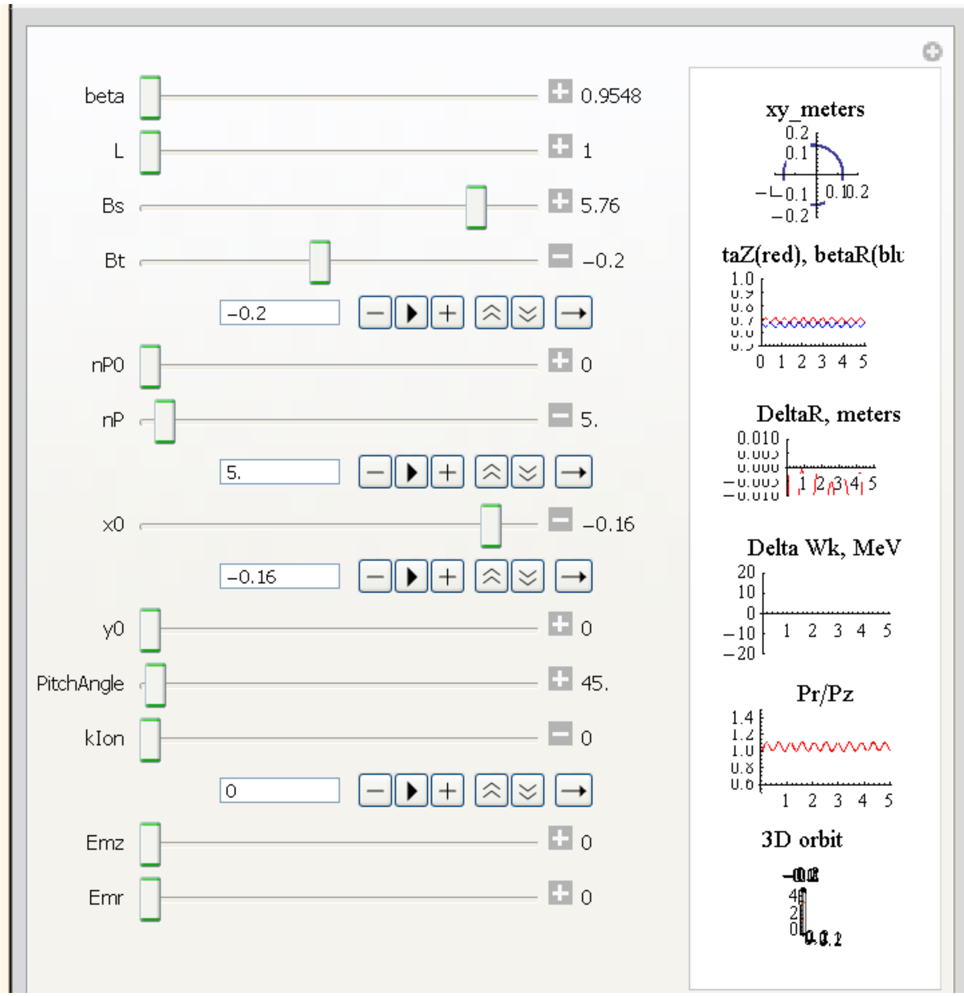
where dW/ds is ionization losses per unit of trajectory length

Simplifications

- **Longitudinal component of helix dipole was assumed $B_z = 0$, since I couldn't find equilibrium orbit with given helix magnet parameters. So, only uniform solenoidal field $B_s = -6.95$ T was taken into account.**
- **Constant gradient uniform accelerating field can be of arbitrary direction in simulations, but it is always synchronous with muons, i.e. longitudinal dynamic and cooling were not considered.**
- **These simplification shouldn't change much the results for transverse motion (Shadwick et al., PAC99, TUP101, MODELING THE MUON COOLING CHANNEL USING MOMENTS)**

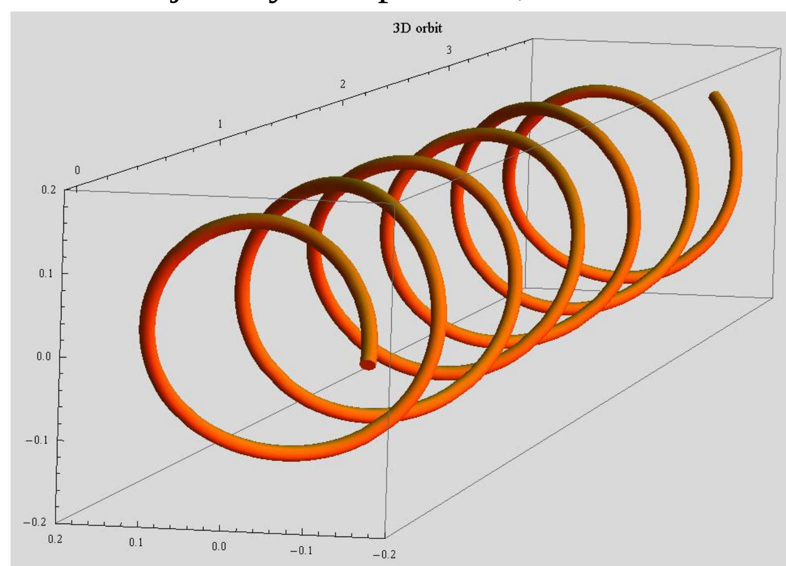
Mathematica code

The equations of motion for a muon are solved numerically with *Mathematica*



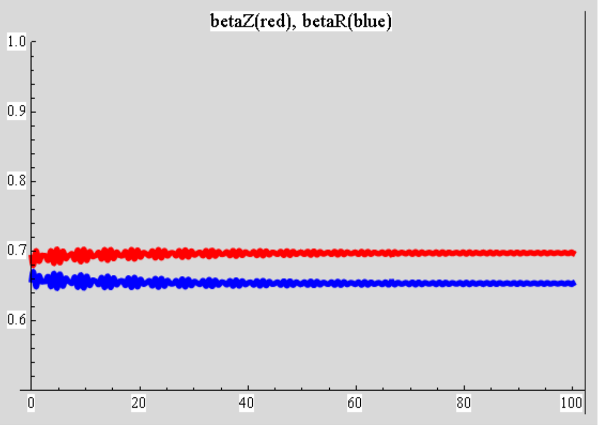
This GUI allows to manipulate parameters and obtain the results in various forms:

3D trajectory – impressive, but useless

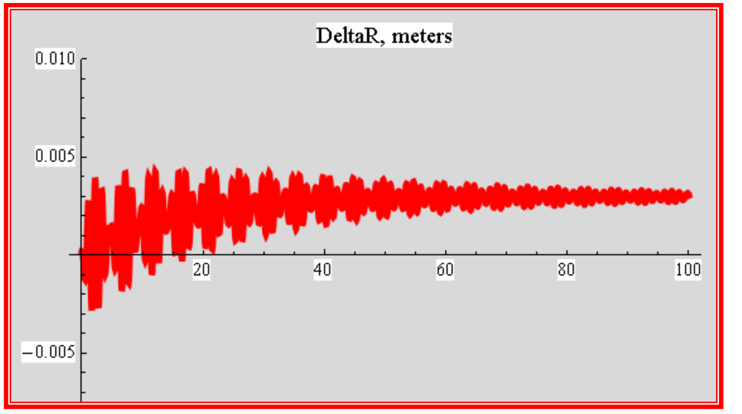


Mathematica output

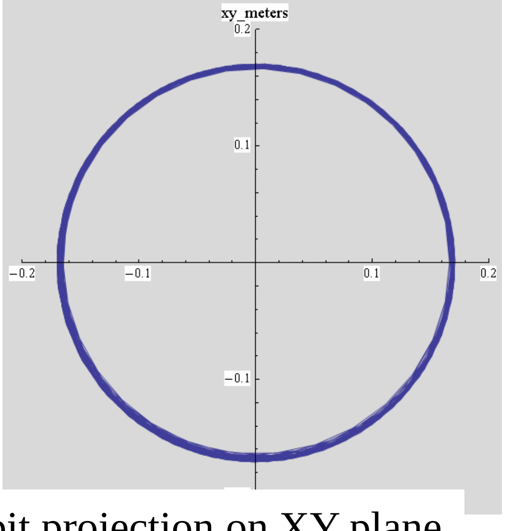
Some examples for non-equilibrium particle (evolution over 100 helix periods)



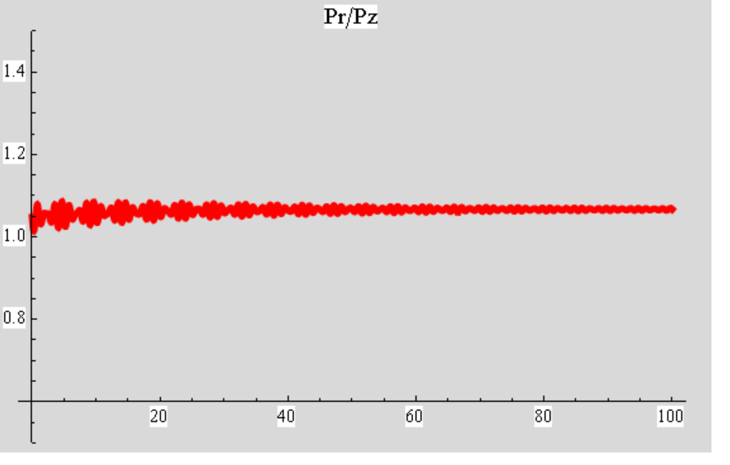
Beta_z, beta_r vs helix period



Deviation from equilibrium orbit



Orbit projection on XY plane



Pr/Pz ratio

Reference orbit

Reference orbit with parameters (close to design):

Muon energy – 250 MeV

B_s = - 6.97 T

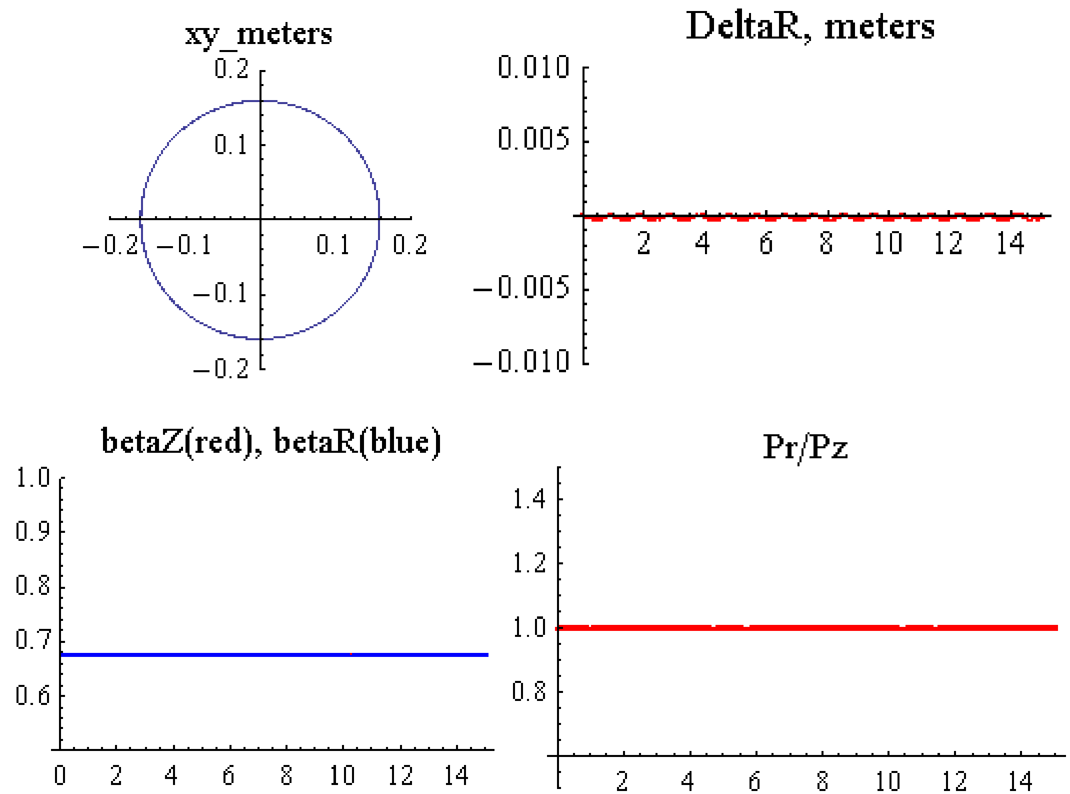
B_τ = 1.73 T

Orbit radius = 0.16 m

X initial = 0.16 m

Y initial = 0

Initial arctg(Pr/Pz) = 45°



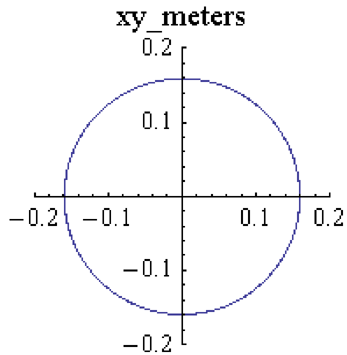
Orbits with ionization and **E**

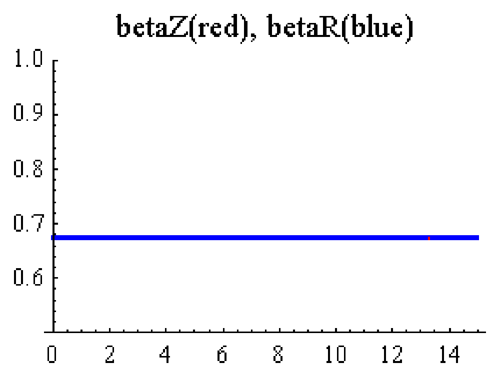
dW/ds = 30 MeV/m

Er = 20.9 MeV/m

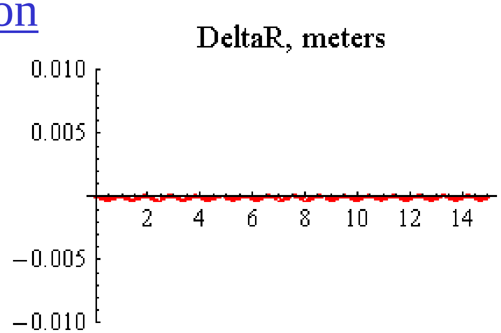
Ez = 20.9 MeV/m

Arctg(Er/Ez)=45°





Co-linear **E** produces almost perfect compensation

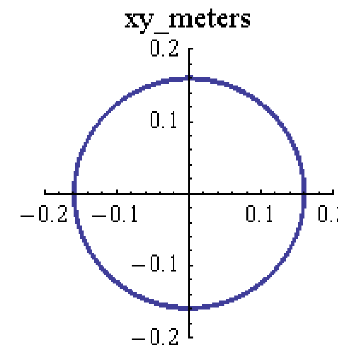


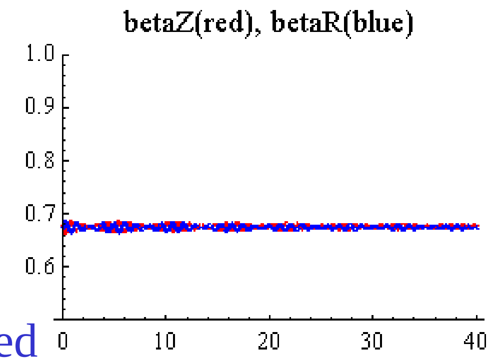
dW/ds = 30 MeV/m

Er = 0

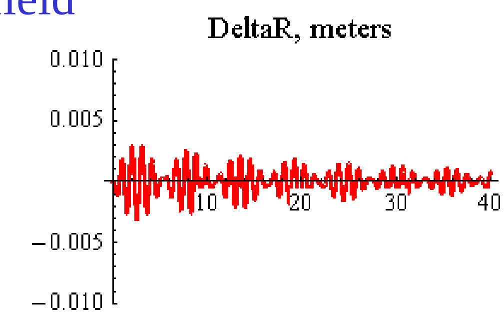
Ez = **41.9** MeV/m

Arctg(Er/Ez)=0



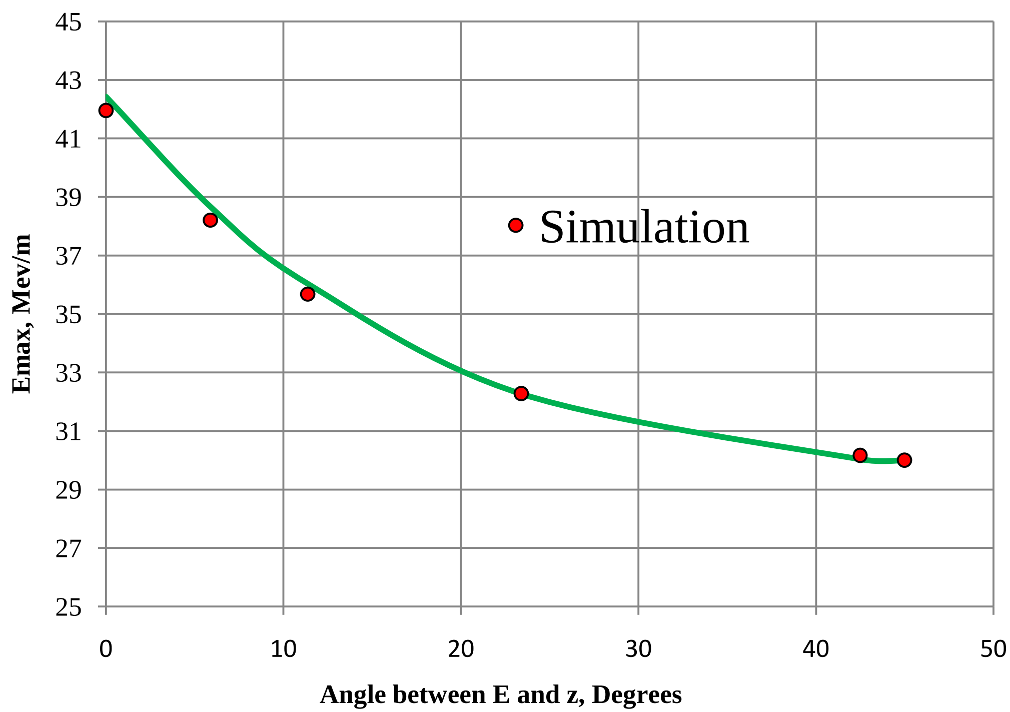


Ez only generates betatron oscillations, though they are cooled down. But electric field amplitude is higher

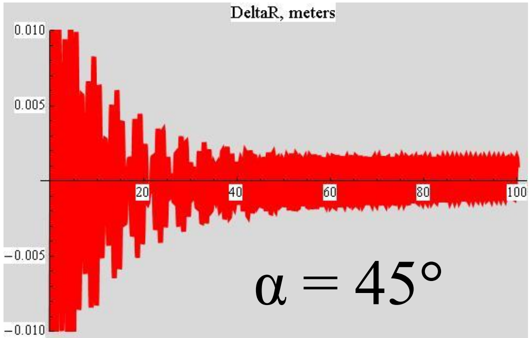


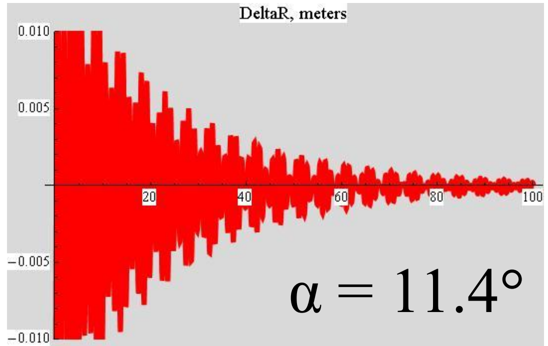
E_{max} required for compensation

Transverse motion is stable up to Arctg(Er/Ez)=45°, but accelerating field amplitude required for compensation of ionization losses is E_{max} = (dW/ds)*cos(α), where α is an angle between **E** and z.



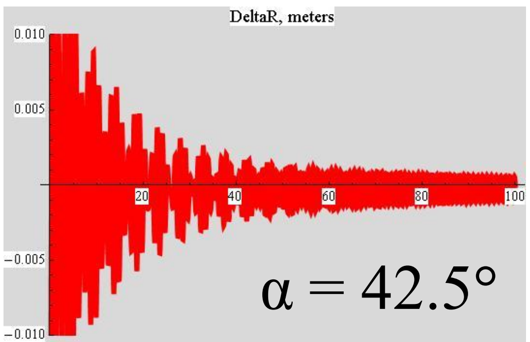
Cooling effectiveness

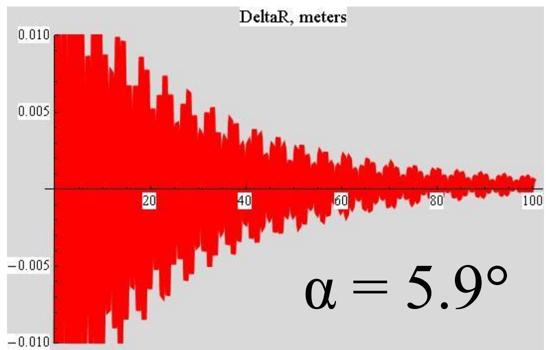


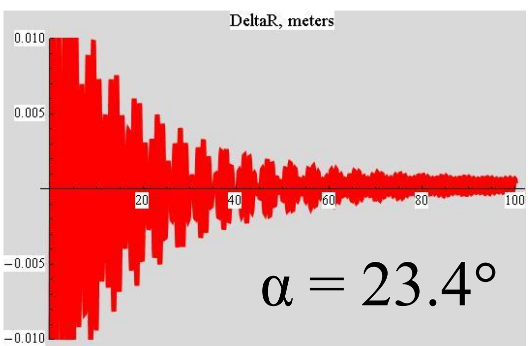


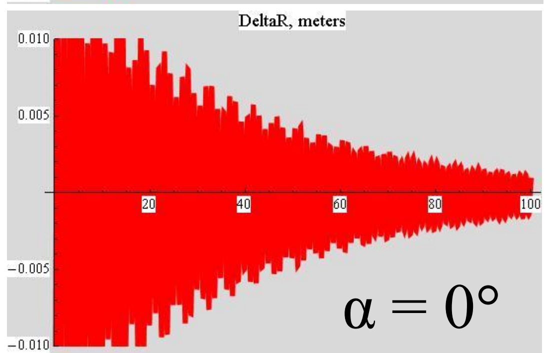
Deviation from reference orbit:

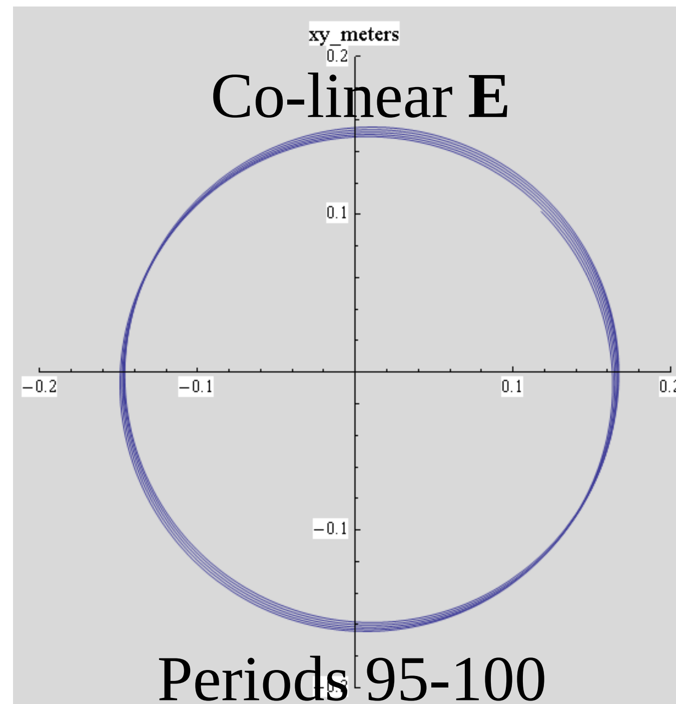
Δr=1cm, $\Delta(Px/Pz)$=1.03











CONCLUSIONS

- **If E is not co-linear with muon orbit higher gradient is required to compensate ionization losses following simple vector consideration**
- **Cavity orientation with E_z component only is less effective, since a halve of RF power goes to transverse motion energy and eventually to heating of hydrogen.**
- **It was expected that the cavity orientation with E co-linear with muon orbit would be the most effective. But the case demonstrated residual oscillations which are not understood.**