On orientation of accelerating cavities in HCC

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Talk Outline

• Motivation
• CST model of helical dipole.
• Analytical model - simplifications. Equations of particle motion
• Solving the equations in Mathematica
• Ionization losses and energy gain. Equilibrium orbits. RF field amplitude
• Impact of RF cavity orientation on cooling.
Ionization reduces $p_x$, $p_y$ and $p_z$, RF cavities restore $p_z$, reducing $p_{x,y}/p_z$ and thus transverse emittance.

In HCC equilibrium orbit is a helix, and accelerating field is not collinear with it. Does the transverse kick matter? Should we try to reduce it?
CST helical dipole model

Period $L = 1\, \text{m}$
Bore $R = 0.3\, \text{m}$
$B_s = -6.95\, \text{T}$
$B_{\tau} = 1.62\, \text{T}$
Orbit radius $= 0.16\, \text{m}$

V. Kashikhin et al, EPAC08, WEPD015

Figure 1: High-field helical dipole with straight solenoid (a quarter removed for clarity).
Unfortunately it’s not practical to use CST Particle Studio directly because of number of reasons. Two of them are distorted field at the ends of the magnet and $B_z$ component in the vicinity of equilibrium orbit. The CST model was used to verify the following approximation of helical dipole fields

(J. P. Blewett and R. Chasman, J. App. Phys. 48, 2692(1977)):

\[
B_x \approx -B_0 \left[ 1 + \frac{k^2}{8} \left( 3x^2 + y^2 \right) \right] \sin kz - \frac{k^2}{4} xy \cos kz,
\]

\[
B_y \approx B_0 \left[ 1 + \frac{k^2}{8} \left( x^2 + 3y^2 \right) \right] \cos kz - \frac{k^2}{4} xy \sin kz,
\]

\[
B_z \approx -B_0 k \left[ 1 + \frac{k^2}{8} \left( x^2 + y^2 \right) \right] \left[ x \cos kz + y \sin kz \right],
\]

where $k = 2\pi/L$
Comparison of fields

Graph 1: Comparison of By/Be centers

Graph 2: Comparison of Bz/Be centers
Equations of motion

In Cartesian coordinates we have for vector components:

\[
\frac{d}{dt} (\gamma m \dot{x}) = q \left( E_x + \dot{y} B_z - \dot{z} B_y \right) + \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}} \left( \frac{dW}{ds} \right)
\]

\[
\frac{d}{dt} (\gamma m \dot{y}) = q \left( E_y + \dot{z} B_x - \dot{x} B_z \right) + \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}} \left( \frac{dW}{ds} \right)
\]

\[
\frac{d}{dt} (\gamma m \dot{z}) = q \left( E_z + \dot{x} B_y - \dot{y} B_x \right) + \frac{\dot{z}}{\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}} \left( \frac{dW}{ds} \right)
\]

where \(dW/ds\) is ionization losses per unit of trajectory length.
Simplifications

• Longitudinal component of helix dipole was assumed $B_z = 0$, since I couldn’t find equilibrium orbit with given helix magnet parameters. So, only uniform solenoidal field $B_s = -6.95$ T was taken into account.

• Constant gradient uniform accelerating field can be of arbitrary direction in simulations, but it is always synchronous with muons, i.e. longitudinal dynamic and cooling were not considered.

• These simplification shouldn’t change much the results for transverse motion (Shadwick et al., PAC99, TUP101, MODELING THE MUON COOLING CHANNEL USING MOMENTS)
The equations of motion for a muon are solved numerically with *Mathematica*

This GUI allows to manipulate parameters and obtain the results in various forms:

3D trajectory – impressive, but useless
Some examples for non-equilibrium particle (evolution over 100 helix periods)

1. Beta\(_z\), beta\(_r\) vs helix period
2. Deviation from equilibrium orbit
3. Orbit projection on XY plane
4. Pr/Pz ratio
Reference orbit with parameters (close to design):
Muon energy – 250 MeV
$B_s = -6.97$ T
$B_{\tau} = 1.73$ T
Orbit radius = 0.16 m
X initial = 0.16 m
Y initial = 0
Initial $\arctan(Pr/Pz) = 45^\circ$
Orbits with ionization and \( E \)

\[ \frac{dW}{ds} = 30 \text{ MeV/m} \]

\[ Er = 20.9 \text{ MeV/m} \]

\[ Ez = 20.9 \text{ MeV/m} \]

\[ \text{Arctg}(Er/Ez)=45^\circ \]

Co-linear \( E \) produces almost perfect compensation

\[ \frac{dW}{ds} = 30 \text{ MeV/m} \]

\[ Er = 0 \]

\[ Ez = 41.9 \text{ MeV/m} \]

\[ \text{Arctg}(Er/Ez)=0 \]

Ez only generates betatron oscillations, though they are cooled down. But electric field amplitude is higher
$E_{\text{max}}$ required for compensation

Transverse motion is stable up to $\arctan(E_r/E_z)=45^\circ$, but accelerating field amplitude required for compensation of ionization losses is $E_{\text{max}} = (dW/ds)\cos(\alpha)$, where $\alpha$ is an angle between $E$ and $z$. 

![Graph showing $E_{\text{max}}$ vs Angle between $E$ and $z$, Degrees]
Cooling effectiveness

Deviation from reference orbit:
$\Delta r = 1\text{cm}$, $\Delta(P_x/P_z) = 1.03$

$\alpha = 45^\circ$
$\alpha = 11.4^\circ$
$\alpha = 42.5^\circ$
$\alpha = 5.9^\circ$
$\alpha = 23.4^\circ$
$\alpha = 0^\circ$

Periods 95-100
CONCLUSIONS

- If E is not co-linear with muon orbit higher gradient is required to compensate ionization losses following simple vector consideration.
- Cavity orientation with $E_z$ component only is less effective, since a halve of RF power goes to transverse motion energy and eventually to heating of hydrogen.
- It was expected that the cavity orientation with E co-linear with muon orbit would be the most effective. But the case demonstrated residual oscillations which are not understood.