Geometry and construction of mirrors for CKOV1

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1. Frame of reference.
Assume the MICE frame of reference:
- with the Z-axis, positive downstream, along the beam,
- with the Y-axis vertical, positive upwards, and
- with the X-axis in the horizontal plane and oriented such as to get a right-handed frame.

We assume the origin is located on the Z-axis at the center of the upstream face of the aerogel radiator.

In addition, let us define two orthogonal planes containing the z-axis and oriented at ±45 degrees with respect to the YZ- and XZ planes. In the drawing below, they are also defined by the OU and OV lines (dashed) together with the Z-axis.

![Figure 1. The MICE frame of reference.](image)

2. Photomultipliers
CKOV1 has a fourfold symmetry around the beam axis OZ. The sensitive areas of the four "equivalent plane" photocathodes are located at y = 303.08 mm and x = 303.08 mm.

Let us start by considering the PMT located "at 12 o'clock" (above the horizontal XOZ plane). The diameter of its supporting ring is 220 mm and its center C has coordinates (X=0; Y = 303.08 mm; Z = 169.5 mm) (Figure 2).

3. Aerogel
The diameter of the aerogel radiator is 440 mm.

In order to catch all photons from the radiator, let us make a mirror whose opening in the XY-plane largely encompasses the whole radiator (with a large safety margin). The opening diameter of the mirror is then taken to be 556.82 mm. It is centered at the origin O and lies in the XY-plane.

The sensitive area of the PMT and the opening diameter of the mirror are represented in Figure 2.
The theoretical mirror is obtained by construction of a cone, which passes through the two circles represented in Figure 2.

The circles are in two orthogonal planes. The cone has circular cross sections only for cuts perpendicular to the Y- and Z-axes. All other cross sections have elliptical shapes, in particular those perpendicular to the axis (which is not an axis of symmetry!).

The position of the apex A of the cone is $[X=0; Y = 433.981 \text{ mm}; Z = 351.985 \text{ mm}]$. 
The next operation is to impose fourfold symmetry around the Z-axis.

Let us now cut the cone by three planes
- an horizontal plane passing through point C.
- two orthogonal planes defined by UOZ and VOZ (at ±45 degrees with respect to the X- and Y-axes). The result is shown on Figure 3.

![Figure 3. Conical reflecting element obtained by three cuts (see text).](image)

The remaining conical element is then repeated by three successive 90-degrees rotations around the Z-axis to obtain the final device shown in Figure 4.

![Figure 4. The whole mirror elements for CKOV1.](image)
5. Principle of the mechanical construction.

The conical reflective element represented on Figure 3 is symmetric with respect to the YZ-plane. We will illustrate the construction of half of it shown on Figure 5.

![Figure 5. Symmetric half of a single reflective element.](image)

We will unfold (flatten or develop) the conical surface sketched on figure 5. a) the first step is to define 15 equidistant points on the left-half (positive x-values) of the circle of center O: these are the green crosses on figure 6. From each of these points we draw a straight line to its neighbor and another one towards the apex A. We thus obtain 15 triangles, which are reconstructed in turn on a common plane.

![Figure 6. Replacement of the conical surface by triangular facets.](image)
The result is shown in figure 7. It is the development (unfolding) of a cone of apex A passing through the green crosses of figure 6.

b) The conical surface still needs to be cut by the plane of circle C and a plane V inclined at 45 degrees with respect to the axes X and Y. The final development is shown in figure 8.
The curved line EF corresponds to the cut at 45 degrees; the line DH is due to the curvature by the plane of circle C.

The conical element adapted to a photomultiplier (figure 3) is thus constructed by assembling a folded sheet of shape DEFGH and its symmetric partner about the line AHG.

In a plane coordinate system centered at A, the coordinates of the points defining the shape DEFGH are given in table 1.

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Table 1. Coordinates for the full mirrors.

6. Constraints for the construction.

Besides the mechanical construction, the mirrors shown in Fig.4 have to be aluminised. This operation imposes strong constraints on the mirrors themselves and on their supporting structures:

a. Bending and gluing the plastic sheets shown in Figs. 7-8 after aluminium deposition is dangerous as the manipulations will inevitably spoil the sensitive reflective surfaces.

1) It implies that the plastic sheets have to be bent and glued on the final supporting structures before the evaporation process.

2) It then follows that these structures cannot be constructed from lightweight honeycomb panels as they would preclude getting the high vacuum requested during the evaporation.

We plan to construct the supporting structures from 10-mm thick aluminium plates.

b. The aluminium deposition process is performed in a large vacuum tank and the aluminium vapour reaches the plastic sheets about perpendicularly to their surfaces.

It implies that the process cannot be performed on the inner surfaces of the whole assembly represented in Fig. 4 in the sense that there are re-intrant surfaces, which would not be covered by the aluminium.

The whole assembly then needs to be cut in halves by a vertical XY-plane to avoid the re-intrant parts. Each half will be separately constructed and aluminised before insertion in the main vessel.
The cut cannot be done by a vertical plane passing through the point C (center of the photocathodes) as it would then also cut the central reflecting tip T of the mirrors. The XY-plane then passes through the point K, 30-mm upstream from C (Fig. 9).

Figure 9. The mirror structure cut in two halves by an XY-plane passing through the point K. The upstream part is represented in dark green and the downstream half in red.

7. The supports of the mirrors.

As written above, the supporting structure is built from 10-mm thick aluminium plates.

1) the support of the upstream mirrors
   Its construction does not present much difficulties. The reflecting plastic sheets are made of 3-mm thick Lexan (polycarbonate) or PMMA (Lucite). They can be thermally shaped before being glued to the edges of the support. The proposed glue is a silicone compound Rhodorsil and a primer.
   The PMT-ends of the structure are equipped with properly shaped coupling rings matched to the photocathode rings of the PMT. Tapered edges allow for the appropriate centering of the PMT.
   The shallow shape of the mounted upstream mirror eases the deposition of the reflecting layers (Fig. 10).
2) the support of the downstream mirrors

Contrary to the ring-shaped upstream part, the downstream reflective surfaces cover the whole surface of CKOV1. Consequently the downstream support should avoid any material which would increase the material budget along the beam line.

For this reason, the downstream mirrors should be constructed from PMMA sheets 3-mm thick. This material easily allows reliable gluing to assemble a solid self-supporting 3D reflecting structure (Fig. 11). The best glue is cheaply obtained from PMMA dissolved in chloroform CHCl₃.
The unfolding of the upstream and downstream mirrors is sketched in Figure 12. The coordinates and dimensions are listed in Tables 2 (upstream) and 3 (Downstream).

Fig.12. Shapes of the unfolded upstream (left) and downstream mirrors (right). Bending 4 left pieces and four right pieces and gluing them on the supports described above reconstruct the whole mirrors surfaces for CKOV1.

Table 2. Coordinates of the points defining the unfolded upstream mirrors.

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Table 3. Coordinates of the points defining the unfolded downstream mirrors.

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The origin of the coordinates is the apex A of the conical surfaces.

The coordinates are available as an Excel file. For the CNC milling machines, it is also possible to deliver the radii of curvature at all points along the edges of the unfolded cones.