ABSTRACT

The Muon Ionization Cooling Experiment (MICE) has three 350-mm long liquid hydrogen absorbers to reduce the momentum of 200 MeV muons in all directions. The muons are then re-accelerated in the longitudinal direction by 200 MHz RF cavities. The result is cooled muons with a reduced emittance. The energy from the muons is taken up by the liquid hydrogen in the absorber. The hydrogen in the MICE absorbers is cooled by natural convection to the walls of the absorber that are in turn cooled by helium gas that enters at 14 K. This report describes the MICE liquid hydrogen absorber and the heat exchanger between the liquid hydrogen and the helium gas that flows through passages in the absorber wall.

INTRODUCTION

The muon ionization cooling experiment (MICE) uses liquid hydrogen absorbers to absorb energy from a beam of 200 MeV/c muons. This energy absorbed is in both the longitudinal and the transverse directions. Energy is put back into the muons in the longitudinal direction. If the heating due to scattering is less than the cooling through absorption, there is net cooling of the muon beam. In order to minimize scattering, liquid hydrogen or liquid helium is used as an absorber. In addition, the absorber is put into a two-coil solenoid that generates a cusp shaped magnetic field. The magnet that generates this magnetic field is described in references [1] and [2]. The MICE absorber is buried in the bore of a superconducting gradient solenoid magnet.

Heat from the beam and heat leaks into the absorber is removed by a stream of helium gas that enters the absorber shell at 14 K. The exit temperature for the gas depends on the total amount of heat that is to be put into the absorber and the mass flow of the helium gas that passes through the heat exchanger in the absorber shell. The sub-cooled hydrogen that is in the absorber body is cooled by natural convection heat transfer to the absorber wall. The heat exchange surface on both sides of the hydrogen vessel wall is extended in order to improve the heat transfer between the sub-cooled liquid in the absorber and the helium flowing in the heat exchanger on the outside of the absorber. The absorber system is also designed so that the absorber can contain sub-cooled liquid helium in place of the hydrogen. Figure 1 shows an end view of the hydrogen absorber inside of the superconducting solenoid warm bore.
FIGURE 1. A view of the MICE focusing solenoid and liquid hydrogen absorber system as seen from the piping end. The absorber is designed to slip out of the end of the solenoid magnet once the pipes are disconnected. All that one sees of the absorber is its vacuum vessel and safety window.

THE HEAT TRANSFER MODEL

The absorber that is shown in Figure 1 consists of a vacuum vessel and the absorber body that contains about 20 liters of liquid hydrogen. The vacuum vessel prevents oxygen from freezing on the absorber body and or on any line that is at a temperature that is below the triple point temperature for oxygen (~54 K). The absorber vacuum vessel is also kept cold. Its minimum temperature must be above 55 K. The nominal design temperature for the absorber vacuum vessel is about 77 K. The 0.35-mm thick safety window temperature depends on the emissivity of the window surface. The absorber is mounted inside the warm bore (~300 K) of the focusing magnet. The connection from the magnet warm bore to the absorber is made through a low heat leak support. The absorber body and 0.035-mm thick hydrogen window are nominally at 20 K (5.0 K if the absorber contains liquid helium). The connection from the absorber to the vacuum vessel is made through a pair of low heat leak stainless steel support tubes. Figure 2 shows a thermal schematic of the absorber system.
FIGURE 2. A schematic representation of the liquid hydrogen absorber in its vacuum vessel that is mounted in the 300 K bore of the MICE focusing solenoid. Note heat comes into the absorber by conduction, thermal radiation, beam heating, and a heater. Heat leaves the absorber via the helium stream.

Figure 2 shows the sources of heat to the absorber vacuum vessel. The conduction heat $Q_C$ enters via the low heat leak supports that connect the absorber vacuum vessel to the 300 K warm bore tube of the focusing magnet. Most of the radiation heat transfer $Q_R$ to the absorber is from the 300 K to the thin absorber safety windows. The rest of the absorber vacuum vessel should have about 10 layers of MLI to reduce radiation heat transfer to the rest of the vacuum vessel (to < 1 watt). Heat transfer from the absorber vacuum vessel to the absorber body is predominately by conduction through the absorber cold mass supports. There is a small amount of radiation heat transfer between the safety windows and the hydrogen windows. Virtually all of the heat transferred into the absorber body from the absorber vacuum vessel is transferred directly to the helium gas flowing through the body heat exchanger. There are two dominant sources of heat into the liquid in the absorber. They are heating from the beam $Q_B$ and heating due to a heater $Q_H$ at the bottom of the absorber that may be used to ensure that natural convection occurs in the absorber. The total heat into the absorber $Q_T$ is removed by helium gas (two-phase liquid helium in the case of a liquid helium absorber) that enters at temperature $T_1$ (~14 K) and leaves the absorber at temperature $T_2$. The heat transfer equation for the absorber can be represented by the following expression

$$Q_T = Q_C + Q_R + Q_B + Q_H = m_{He} \Box H = m_{He} C_p [T_2 - T_1]$$

where $m_{He}$ is the helium mass flow through the absorber; $\Box H$ is the enthalpy change of the helium stream; $C_p$ is the specific heat per unit mass of the gas stream. Note $\Box H$ can also be the enthalpy change along a stream of two-phase helium at 4.3 K, when the absorber body contains sub-cooled liquid helium at about 5 K. The specific heat of helium gas in the temperature range from 14 K to 20 K is about 5200 J kg$^{-1}$ K$^{-1}$. For boiling helium, the maximum enthalpy change from the saturated liquid line to the saturated vapor line is of the
order $19600 \text{ J kg}^{-1}$. For a given heat load into the absorber, the mass flow of helium into the liquid helium absorber is about a factor of 3.6 lower than for a liquid hydrogen absorber, where the helium is a gas.

The heat transfer into the helium stream and through various components of the absorber can be represented by a network that is similar to an electrical network. In the thermal network, temperature $T$ is analogous to voltage and heat flow $Q$ is analogous to current in an electrical network (see references [3], [4], and [5]). The thermal resistances are analogous to electrical resistors. For a given set of temperatures and a given total heat flow, the network has a solution for the resistances that will yield the desired result. An example of the thermal network for the absorber system when it is filled with liquid hydrogen is shown in Figure 3.

In order to solve the network shown in Figure 3, one must iterate the solution. To determine the value of $R_{t1}$ and $R_{t2}$, one must assume that $Q_c = 20 \text{ W}$ and one must assume that $Q_r = 0$. When $Q_r = 0$, the temperature $T_1$ must be about 60 K. This means that $R_{t1} = 4R_{t2}$. When one puts $Q_r$ back into the equation $T_1$ is about 77 K. Because of the nature of the radiation heat transfer equation $R_r \gg R_{t1}$ and $R_{t2}$ must be $\gg R_{vin}$. The equations used to calculate the resistances in the network shown in Figure 3 are given in reference 5. The details of solving the network are also given in reference 5.

In order to achieve a temperature $T_r = 20 \text{ K}$ in the absorber when the average temperature of the helium stream $T_{He} = 16 \text{ K}$ certain values of the thermal resistances $R_{c1}$ and $R_{c2}$ must be obtained. Once one knows the desired value for the thermal resistances on the inside of the absorber body and the inside of the helium tube, one can design a heat exchanger that will have the proper $U$ factor to transfer the heat from the absorber to the helium gas. The point is that a network solution can lead to a physical solution to the absorber heat transfer problem. The network tells one how to design the vacuum vessel supports, the support between the absorber and the vacuum vessel and the amount of heat transfer surface needed to transfer 100 W to the helium stream that enters at 14 K and leaves at 18 K (so $T_{He} = 16 \text{ K}$). A similar network analysis as applied to the case where the hydrogen absorber body is filled with sub-cooled liquid helium at 5.1 K and the absorber case contains boiling two-phase helium at 4.3 K. The network analysis for this case is shown in reference 5 for a total heat flow $Q_t = 47 \text{ W}$.

![Figure 3](image)

**FIGURE 3.** The thermal network for the MICE absorber when it is filled with liquid hydrogen. The network shown assumes a conductive heat flow of 19.6 W and a radiation heat flow of 10.6 W. The total beam heating plus heater heating is 70 W. Thus, the helium gas entering at 14 K removes 100 W.
HEAT TRANSFER IN THE ABSORBER BODY

A number of finite element free convection heat transfer calculations have been done. The results of these calculations show that 100 W can be transferred from the hydrogen to a cold wall at 14 to 16 K. The analysis shows that the top of the absorber is the predominant place where the heat transfer by free convection occurs. The heat transfer coefficient at the bottom of the absorber is poor, because there is little or no free convection flow on that surface. An experiment in Japan showed that over 100 W can be transferred from 27 K neon to helium in a cylindrical body that is 280 mm in diameter [6].

Figure 4 illustrates an alternative scheme where the free convection flow next to the heat transfer surface is ducted in two dimensions. This results in a larger effective heat transfer surface and a larger value of UA from the hydrogen side to the forced flow helium side of the heat exchanger within the absorber body. When one ducts the flow, the hydrogen will flow down the wall. The helium must flow up the wall forming a counter-flow heat exchanger.

The duct configuration will be more complex than shown in Figure 4, in order to direct the hydrogen streamlines and prevent the formation of eddies that will increase the pressure drop on the hydrogen side. The buoyancy forces in the heated hydrogen must balance the pressure drop. More study is needed on a three dimensional absorber with windows.

FIGURE 4. The flow pattern for the hydrogen and helium in an absorber that is ducted to maximize the total heat transfer coeffcient UA from the liquid hydrogen to the helium gas in the absorber wall.
Two factors determine the performance of the heat exchanger between the hydrogen and the helium gas. The first factor is the area of the heat exchanger surfaces on both sides of the heat exchanger. The second factor is the type of heat exchanger that is used.

In order for the hydrogen temperature in the absorber to be 20 K when the helium enters the heat exchanger at 14 K and leaves the heat exchanger at 18 K, the heat exchange surface on both sides of the heat exchanger must be extended. If the heat exchanger surface were not extended, the heat transfer area would be about 0.175 m$^2$ on both sides of the heat exchanger. Fins on both sides of the heat exchanger will extend the surface on both sides over a factor of more than two. It is quite reasonable to design the absorber such that the heat exchanger area on both sides of the heat exchanger ranges from 0.41 to 0.44 m$^2$. The cost of extending the heat transfer surface is the extra $\Delta T$ required to transfer the heat along the fins.

The heat transfer studies that were done in reference 5 showed that the $\Delta T$ across the heat exchanger is dominated by the $\Delta T$ on the forced flow helium side when the helium mass flows are low. When one increases the mass flow on the helium side of the heat exchanger, the $\Delta T$ across the heat exchanger becomes dominated by the $\Delta T$ on the free convection side (the hydrogen side) of the heat exchanger.

The other key element in the design of the heat exchanger is the direction of the helium flow relative to the hydrogen flow down the absorber walls. The helium must flow upward in the absorber case so that a counter-flow heat is exchanger is formed. This is the type of heat exchanger that is shown in Figure 4. Counter flow heat exchangers are commonly used in cryogenic devices, because a counter flow heat exchanger can have the coldest temperature of the warm fluid colder than the warmest temperature of the cold fluid. Other types of heat exchangers such as parallel flow or cross flow heat exchangers do not exhibit this feature. Ducting of the flow on the hydrogen side makes the use of a counter flow heat exchanger more attractive, because the entire inner surface of the absorber can be a part of an efficient heat transfer surface.

Figure 5 shows a schematic cross-section of the absorber body and the heat exchanger between the liquid hydrogen and the cold helium gas. The area on both sides of the heat exchanger is shown to be the same. This is a reasonable compromise for a system that will be tested over a range of $Q_T$ from 30 W to 150 W. The absorbers for MICE are designed for a heat flow $Q_T = 100$ W, but the actual heat flow during the experimental runs is expected to be much closer to 50 W. If one were to design the MICE absorber for the expected absorber heat load, the extended heat exchanger surface shown in Figure 5 would probably not be needed.

![Figure 5](image.jpg)

**FIGURE 5.** A cross-section of the MICE hydrogen absorber showing the extended heat transfer surface in the case heat exchanger. The heat exchanger shown has an approximate heat transfer area of 0.42 m$^2$. The absorber shown has bolted windows; welded windows would work as well.
The expansivity of liquid hydrogen at 1 bar from 16 to 20 K is about 0.02 K⁻¹ [7]. Changes in hydrogen density of 5 percent are allowed in MICE absorbers. This means that the maximum temperature variation within an absorber can be as much as 2.5 K. The flow equations and the heat transfer equations for the hydrogen flow case shown in Figure 4 are given in Ref. [5]. From these equations, one can calculate the maximum temperature of the sub-cooled hydrogen as a function of the total heat into the system $Q_t$ and the mass flow rate of the helium gas $m_{He}$ entering the absorber body from the bottom. The heat transfer area for the extended heat exchanger is about 0.4 m² on both sides of the heat exchanger. Figure 6 presents the results of the heat transfer calculations for values of total heat into the absorber of 50 W, 100 W, 150 W, and 200 W as a function of the mass flow of the helium gas stream. Figure 6 plots the peak average temperature of the liquid hydrogen as a function of the total heat into the absorber and the mass flow of the helium stream entering the absorber at 14K.

The studies done in Ref. [5] show two additional features of this type of heat exchanger problem. The first is the mass flow and average $\bar{T}$ of the fluid on the free convection side of the heat exchanger will vary approximately as the square root of the heat transferred across the free convection side of the heat exchanger). Second, for a given amount of heat transferred, the $\bar{T}$ of the fluid on the free convection side of the heat exchanger are inversely proportional the square root of the cross-section area of the fluid within the duct. The explanation of this is based on the notion that the driving pressure for the hydrogen flow is proportional to the density change as the hydrogen is heated, which for constant specific heat is proportional to the bulk temperature change in the hydrogen. The pressure drop in the flow circuit (for a short flow circuit) is dominated by the momentum change of the fluid as it changes direction in the duct. (Friction is not a large term in the flow circuit shown in Figure 4.) In the limit where friction is absent, the pressure drop in the fluid entering and leaving the duct is proportional to the mass flow per unit duct area squared. Since the duct pressure drop can not be greater than the driving pressure, the mass flow per unit area is proportional to the square root of the bulk temperature change in the hydrogen. Friction in the ducts further reduces the mass flow through the ducts, but not to a great extent when the ducts are short.
Since to first order the specific heat of the fluid in the absorber body is a constant over the operating temperature range for that fluid, the product of the fluid mass flow and the fluid $\Delta T$ is proportional to the heat being transferred to the wall. This statement is also true for the fluid on the helium side of the heat exchanger.

The heat transfer study done in reference 5 showed that the proposed MICE absorber heat exchanger was adequate for transferring heat from sub-cooled liquid helium in the temperature range from 4.8 to 5.1 K to two-phase helium flowing in the absorber case. This is true provided the sum of $Q_h$ and $Q_L$ is less than 15 W. Because the heat absorption is limited, a liquid helium absorber will not work for a high-intensity muon-cooling channel.

**CONCLUDING COMMENTS**

Free convection heat transfer from the liquid in an absorber to fluid flowing to a cryogenic fluid in the walls of an absorber has been demonstrated by experiment [6]. This report and reference [5] demonstrate that a network analysis can be used to study the heat transfer in a liquid hydrogen absorber system. While free convection heat transfer to the inner surface of the absorber case has been demonstrated, the use of a duct to direct the free convection needs to be demonstrated in a three dimensional system.

It appears that a liquid hydrogen can be cooled using free convection heat transfer to the absorber wall that is cooled by helium gas. Over the range of heat depositions expected in the MICE absorber, the proposed heat exchanger system will work over a range of heat flows into the hydrogen absorber up to 180 W. When the MICE absorber is filled with sub-cooled helium, the absorber can be cooled using two-phase helium flowing in the case. The upper limit on the total heat to be transferred into the absorber is about 40 W for sub-cooled helium temperature of about 5.1 K.

Heat into the liquid hydrogen in the absorber causes the hydrogen to flow. The hydrogen mass flow rate in the absorber and the bulk $\Delta T$ in the hydrogen stream are proportional to the square root of the amount of heat deposited into the absorber liquid. For a ducted absorber surface, the hydrogen mass flow for a given heat flow will be proportional to the square root of the duct cross-section area. For a given heat transfer the $\Delta T$ in the stream will be inversely proportional to the stream mass flow.

**ACKNOWLEDGEMENTS**

The authors acknowledge discussions with Christine Darve of Fermilab and Ed Black and Mary Anne Cummings of IIT concerning various aspects of hydrogen absorbers. This work was performed at the Lawrence Berkeley National Laboratory with the support of the Office of Science, United States Department of Energy under DOE contract DE-AC03-76SF00098.

**REFERENCES**

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