Simulation
Station spacing and tracker volume material

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Measurement principle

Let me remind you basics.

- The tracker measures space points of muon trajectory with five stations

- $p_T$: estimated by finding the radius of the trajectory using five points
  \[ p_T = 0.3BR \]

- $p_z$: estimated from the following relation
  \[
  \tan \theta = \frac{R \Delta \phi}{\Delta z} = \frac{p_T/E \Delta t}{p_z/E \Delta t} = \frac{p_T}{p_z}
  \]
  \[ p_z = 0.3B \frac{\Delta z}{\Delta \phi} \]

Helix track parameters:

- $x = R \sin(\alpha z + \phi_0) + x_0$
- $y = R \cos(\alpha z + \phi_0) + y_0$

\[ \alpha = \frac{\Delta \phi}{\Delta z} \]

$\Delta z_i$
MICE will measure the following muon beam; combination of $p_z$ and $B$

<table>
<thead>
<tr>
<th>$p_z$ (MeV)</th>
<th>140</th>
<th>170</th>
<th>200</th>
<th>240</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$ (T)</td>
<td>2.8</td>
<td>3.4</td>
<td>4.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>

$\sigma_{p_z} = 25$ MeV

In principle three points are enough to constrain a circle ($p_T$ measurement):

- to make one third of a turn in 4T, [25, 30, 35 and 40] cm spacing is required for [140, 170, 200 and 240] MeV
- to make one third of a turn in 2.8/3.4T, 35 cm spacing for 140/170 MeV

Then if we can have the above set of spacing, 25cm-30cm-35cm-40cm, we probably have good performance for entire region of planned momentum.

**But, 25 + 30 + 35 + 40 = 130 cm, is too long.** Try another set.
## Station spacing

Set of station spacing studied:

<table>
<thead>
<tr>
<th></th>
<th>1–2</th>
<th>200</th>
<th>150</th>
<th>300</th>
<th>240</th>
<th>225</th>
</tr>
</thead>
<tbody>
<tr>
<td>100cm</td>
<td>470</td>
<td>250</td>
<td>200</td>
<td>200</td>
<td>220</td>
<td>175</td>
</tr>
<tr>
<td>2–3</td>
<td>200</td>
<td>250</td>
<td>150</td>
<td>300</td>
<td>240</td>
<td>225</td>
</tr>
<tr>
<td>3–4</td>
<td>180</td>
<td>250</td>
<td>300</td>
<td>300</td>
<td>260</td>
<td>275</td>
</tr>
<tr>
<td>4–5</td>
<td>150</td>
<td>250</td>
<td>350</td>
<td>200</td>
<td>280</td>
<td>325</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1–2</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>105cm</td>
<td>400</td>
<td>200</td>
</tr>
<tr>
<td>2–3</td>
<td>350</td>
<td>100</td>
</tr>
<tr>
<td>3–4</td>
<td>100</td>
<td>350</td>
</tr>
<tr>
<td>4–5</td>
<td>200</td>
<td>400</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1–2</th>
<th>200</th>
<th>150</th>
<th>450</th>
<th>100</th>
<th>200</th>
<th>245</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>110cm</td>
<td>350</td>
<td>275</td>
<td>150</td>
<td>450</td>
<td>100</td>
<td>200</td>
<td>245</td>
<td>200</td>
</tr>
<tr>
<td>2–3</td>
<td>300</td>
<td>275</td>
<td>250</td>
<td>350</td>
<td>250</td>
<td>350</td>
<td>265</td>
<td>250</td>
</tr>
<tr>
<td>3–4</td>
<td>250</td>
<td>275</td>
<td>350</td>
<td>200</td>
<td>350</td>
<td>285</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>4–5</td>
<td>200</td>
<td>275</td>
<td>350</td>
<td>100</td>
<td>400</td>
<td>200</td>
<td>305</td>
<td>350</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1–2</th>
<th>225</th>
</tr>
</thead>
<tbody>
<tr>
<td>120cm</td>
<td>225</td>
<td></td>
</tr>
<tr>
<td>2–3</td>
<td>275</td>
<td></td>
</tr>
<tr>
<td>3–4</td>
<td>325</td>
<td></td>
</tr>
<tr>
<td>4–5</td>
<td>375</td>
<td></td>
</tr>
</tbody>
</table>
Input muon beam

In order to study the tracker performance, the following muon beam is used.

- The input muon beam is matched 2.5 $\pi$ mm rad. beam.
- Number of muons generated is 10k for each case.
- Muons simulated from just in front of the tracker
- G4MICE version: Malcolm-demo-T20050208

Then

- The tracker performance is checked with the upstream tracker.
  Tracker volume is evacuated in G4MICE

Track reconstruction code is being updated to fix 'serious' bug
In order to evaluate tracker performance, the following loose selection cut is applied:

- reject tracks with reconstructed value(s) <= -9999.
- select tracks with # of fit points used = 5

Then residual is calculated as; $p(\text{reconstructed}) - p(\text{true})$
Resolution: $p_T$

Need to evaluate how good/bad tracker performance is, with different spacing. Let’s see the error propagation formulae.

$p_T = 0.3BR$

- $\sigma_{pT} = 0.3B\sigma_R$

\[
R = \sqrt{x^2 + y^2} \rightarrow (\sigma_R)^2 = \left( \frac{\partial R}{\partial x} \right)^2 (\sigma_x)^2 + \left( \frac{\partial R}{\partial y} \right)^2 (\sigma_y)^2
\]

\[
= \frac{1}{R^2} (x^2 (\sigma_x)^2 + y^2 (\sigma_y)^2)
\]

$\rightarrow \sigma_{pT} = 0.3B\sigma_x$, when $\sigma_x \approx \sigma_y$

$x = x_i - x_0 \rightarrow \sigma_x = \sqrt{(\sigma_{x_i})^2 + (\sigma_{x_0})^2}$

$\sigma_{pT}$ is proportional to $\sigma_x$

Position resolution:

- $(\sigma_{x_i})^2 = \int_{-d/2}^{d/2} x^2 \frac{1}{d} dx \Rightarrow \sigma_{x_i} = \frac{d}{\sqrt{12}}$
Resolution: \( p_z \)

\[
p_z = 0.3B \frac{\Delta z}{\Delta \phi}
\]

- \[
\sigma_{p_z} = 0.3B \frac{\Delta z}{(\Delta \phi)^2} \sigma_{\Delta \phi}
\]

\[
\Delta \phi = \phi_i - \phi_{i-1} \rightarrow \sigma_{\Delta \phi} = \sqrt{2} \sigma_{\phi_i}
\]

\[
\phi_i = \arctan \frac{y}{x}
\]

\[
(\sigma_{\phi_i})^2 = \left( \frac{\partial \phi_i}{\partial x} \right)^2 (\sigma_x)^2 + \left( \frac{\partial \phi_i}{\partial y} \right)^2 (\sigma_y)^2
\]

Here, \[
\frac{\partial \phi_i}{\partial x} = -\frac{y}{R^2}, \quad \frac{\partial \phi_i}{\partial y} = \frac{x}{R^2}
\]

\[
\sigma_{\phi_i} = \frac{\sigma_x}{R}, \quad \text{when} \ \sigma_x \simeq \sigma_y
\]

\[
x = x_i - x_0 \rightarrow \sigma_x = \sqrt{(\sigma_{x_i})^2 + (\sigma_{x_0})^2}
\]

\[
\rightarrow \sigma_{p_z} = 0.3B \frac{\Delta z}{(\Delta \phi)^2} \sqrt{2} \frac{\sigma_x}{R}
\]

\[
= \frac{1}{\Delta z p_T} \sqrt{2} \sigma_x
\]

\( \sigma_{p_z} \) is proportional to \( p_z^2 \) and inverse proportional to \( \Delta z \) and \( p_T \).
Selection criteria

Fit rms of residual in terms of $p_t$ and $p_z$.
- $\sigma_{p_z}(p_t) = \alpha / p_t$
- $\sigma_{p_z}(p_z) = \beta * p_z^2$
- $\sigma_{p_t} = \gamma$

Fitting procedure:
- put error on each $\sigma_i$ with $\sigma_i/\sqrt{N_i}$
- fit bins with $N_i > 24$

Selection efficiency:
- # of selected / # of generated tracks
  - reconstructed value(s) $\leq -9999$.
  - # of fit points used = 5
- choose the smallest fitting parameters, $\alpha, \beta$ and $\gamma$, with the best selection efficiency.
Each symbol/line corresponds to different total length;
110cm - black; 100cm - red; 105cm - blue; 120cm
The smaller, the better
Each symbol/line corresponds to different total length; 110cm - black; 100cm - red; 105cm - blue; 120cm
The smaller, the better for $\gamma$.

- longer tracker might cause problems; domino effect to other components (vacuum window position)? B-field uniformity?

$\rightarrow$ the best spacing with total length of 110cm (baseline length) is "200-250-300-350"
**Comments**

Comparison between baseline design and the "best" spacing:

$p_z$ resolution compared to baseline design at 4T:

| $p_z$||$p_T$ (MeV) | 140 | 170 | 200 | 240 | 22.5<$p_T$> | 5  | 15  | 25  |
|---|---|---|---|---|---|---|---|---|---|
| 450-350-200-100 | 3.44 | 5.08 | 7.03 | 10.1 | 4.59 | 20.6 | 6.87 | 4.12 |
| 200-250-300-350 | 3.46 | 5.10 | 7.06 | 10.2 | 4.09 | 18.4 | 6.12 | 3.67 |

$p_T$ resolution and efficiency compared to baseline design at 4T:

<table>
<thead>
<tr>
<th></th>
<th>fit with $p_T$</th>
<th>fit with $p_z$</th>
<th>efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>450-350-200-100</td>
<td>1.82</td>
<td>1.83</td>
<td>0.91</td>
</tr>
<tr>
<td>200-250-300-350</td>
<td>1.23</td>
<td>1.23</td>
<td>0.97</td>
</tr>
</tbody>
</table>

⇒ Improvement on $p_T$ resolution
Tracker volume material

The following materials are considered to fill tracker volume:

- vacuum
- helium
- air

Material properties:

<table>
<thead>
<tr>
<th></th>
<th>density</th>
<th>radiation length</th>
<th>condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>vacuum</td>
<td>1.0e-25 g/cm³</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>helium</td>
<td>0.166 mg/cm³</td>
<td>5671 m</td>
<td>293.15K, 1 atm</td>
</tr>
<tr>
<td>air</td>
<td>1.205 mg/cm³</td>
<td>304 m</td>
<td>293.15K, 1 atm</td>
</tr>
</tbody>
</table>

ref; http://pdg.lbl.gov/AtomicNuclearProperties
Multiple Coulomb scattering

\[ \theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} z \sqrt{x/X_0} [1 + 0.038 \log(x/X_0)] \]

accurate to 11% or better for \( 10^{-3} < x/X_0 < 100 \) [PDG]

- \( y_{\text{plane}}(\text{rms}) = \frac{1}{\sqrt{3}} x \theta_0 \)
- \( \theta_{\text{plane}}(\text{rms}) = \theta_0 \)

The worst case:
the window is at the end of solenoid module; thickness of gas from tracker end to window will be about 2.4 m

The best case:
the window is close to the tracker end; thickness 1.3 m
Effect of gas

For 200 MeV/c muon;

<table>
<thead>
<tr>
<th>material</th>
<th>thickness</th>
<th>$\theta_{\text{plane}}(\text{rms}) \text{ rad.}$</th>
<th>$y_{\text{plane}}(\text{rms}) \text{ m}$</th>
<th>$x/X_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>helium</td>
<td>1.3 m</td>
<td>$7.9 \times 10^{-4}$</td>
<td>$6.0 \times 10^{-4}$</td>
<td>$2.29 \times 10^{-4}$</td>
</tr>
<tr>
<td>air</td>
<td>1.3 m</td>
<td>$4.0 \times 10^{-3}$</td>
<td>$3.0 \times 10^{-3}$</td>
<td>$4.28 \times 10^{-3}$</td>
</tr>
<tr>
<td>helium</td>
<td>2.4 m</td>
<td>$1.1 \times 10^{-3}$</td>
<td>$1.5 \times 10^{-3}$</td>
<td>$4.23 \times 10^{-4}$</td>
</tr>
<tr>
<td>air</td>
<td>2.4 m</td>
<td>$5.6 \times 10^{-3}$</td>
<td>$7.7 \times 10^{-3}$</td>
<td>$7.89 \times 10^{-3}$</td>
</tr>
<tr>
<td>a station</td>
<td>1.9 mm</td>
<td>$4.1 \times 10^{-3}$</td>
<td>$4.5 \times 10^{-6}$</td>
<td>$4.5 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

Filling tracker volume with air is close to have another inactive station or worse.

I would recommend to use helium which have less than 10% of $x/X_0$ compared to a station.
Summary & Todo

☑ Effect of station spacing on tracker performance studied
  ➢ Selection criteria for optimal spacing worked out
  ➢ With this study, the best spacing is 20cm-25cm-30cm-35cm.

☑ Effect of helium gas/air in tracker volume studied
  ➢ I would recommend to use helium gas instead of air from my calculations; simulation study will follow

• Redo the analysis after an update of track reconstruction programme